

Today

- Ch. 8 on Rotation

❖ Note, all lectures and pre-lectures posted up as always at <http://www.hunter.cuny.edu/physics/courses/physics100/spring-2016>

❖ Looking ahead: Sep 27, Review (Chs 2,3,4,5,6,7,8)

❖ Sep 30: Midterm

Chapter 8: Rotational Motion

If you ride near the outside of a merry-go-round, do you go faster or slower than if you ride near the middle?

It depends on whether “faster” means

- a faster **linear speed (= speed)**, ie more *distance* covered per second,

or

- a faster **rotational speed (=angular speed, ω)**, i.e. more *rotations or revolutions* per second.

- The *linear* speed of a rotating object is greater on the outside, further from the axis (center), but the *rotational* speed is the same for any point on the object – all parts make the same # of rotations in the same time interval.

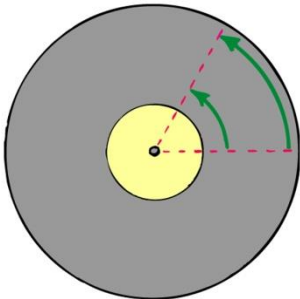
More on rotational vs tangential speed

For motion in a circle, linear speed is often called **tangential speed**

- The faster the ω , the faster the v in the same way (e.g. merry-go-round), i.e. $v \sim \omega$.

directly proportional to

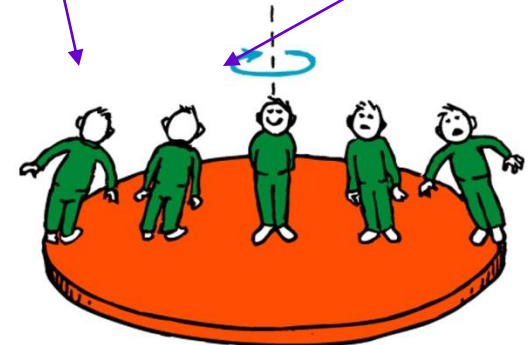
- ω doesn't depend on where you are on the merry-go-round, but v does: i.e. $v \sim r$



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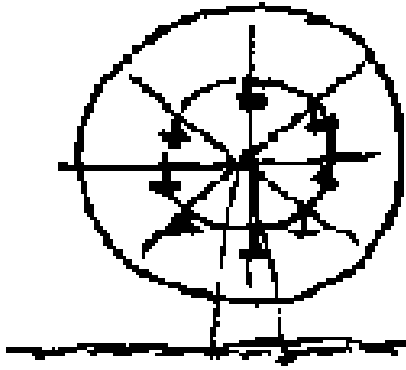
Same RPM (ω) for all these people, but different tangential speeds.

He's got twice the linear speed than this guy.



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Clicker Question



A carnival has a Ferris wheel where the seats are located halfway between the center and outside rim. Compared with a Ferris wheel with seats on the outside rim, your angular speed while riding on this Ferris wheel would be

- A) more and your tangential speed less.
- B) the same and your tangential speed less.
- C) less and your tangential speed more.
- D) None of the above

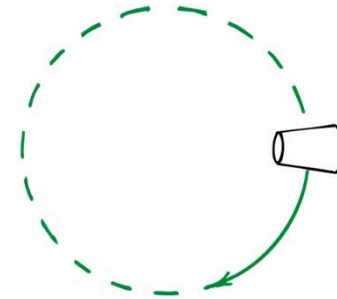
Answer: B

Same # cycles per second i.e. same angular speed, but less distance covered each second, so less tangential speed

Example and Demo: Railroad train wheels

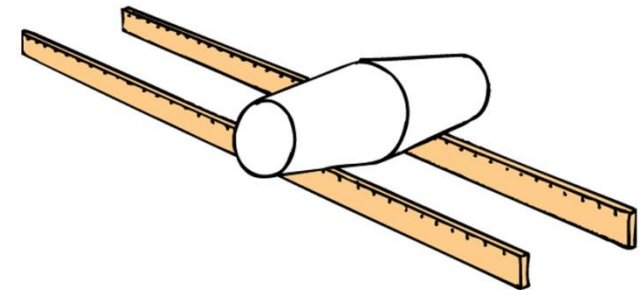
- Model: two tapered cups stuck together and rolling along meter sticks

First note that if you roll a tapered cup along a table, it follows a circle because the wide part moves faster than the narrow part. (Larger v so more distance covered by the wide end).



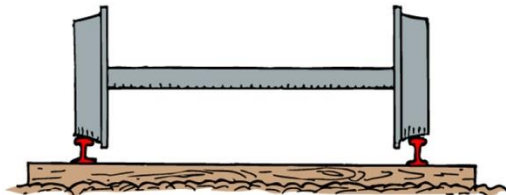
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Now, if you tape two together, at their wide ends, and let them roll along two meter sticks (“tracks”), they will stay stably on the tracks.



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Why? When they roll off center, they self-correct: say they roll to the left, then the wider part of the left cup and the narrower part of the right cup are on the tracks, causing rolling back to the right, since the left cup has larger tangential speed.



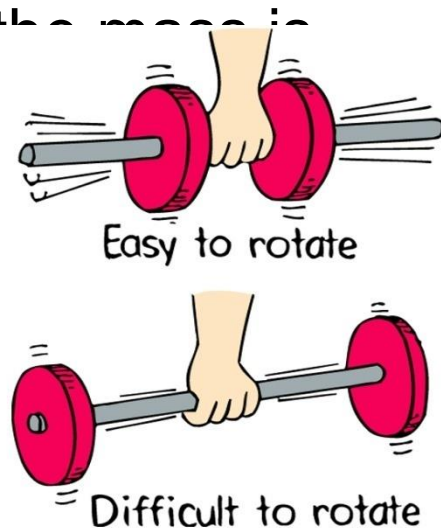
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Railroad wheels act on this same principle!

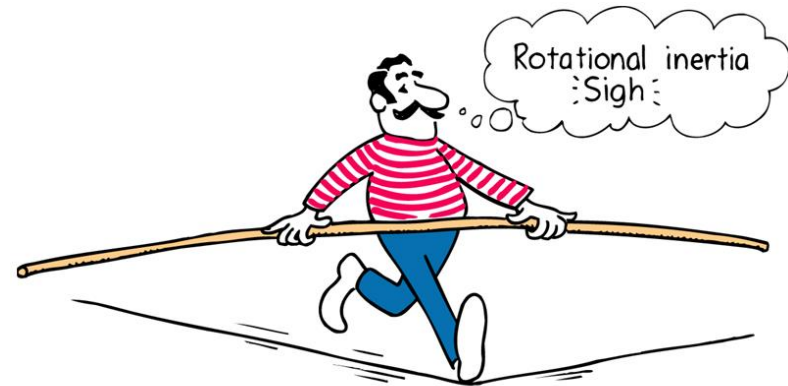
Rotational Inertia

- An object rotating about an axis tends to remain rotating about the same axis, unless an external influence (*torque*, see soon) is acting. (c.f. 1st law)
- The property to resist changes in rotational state of motion is called **rotational inertia**, or **moment of inertia**, I .
- Depends on **mass**, as well as the **distribution of the mass relative to axis of rotation** – largest if the mass is further away from the axis

Eg. **DEMO**: Spinning a pencil with globs of play-doh on it – if the globs are near the ends of the pencil, it is harder to spin, than if the globs are nearer the middle.



Eg. Tight-rope walker carries a pole to increase his rotational inertia - if he starts to wobble, the pole starts to rotate but its inertia resists this, so the tight-rope walker has time to adjust balance and not rotate and fall.



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Better balance (more rotational inertia) if pole is longer and has weights at the ends.

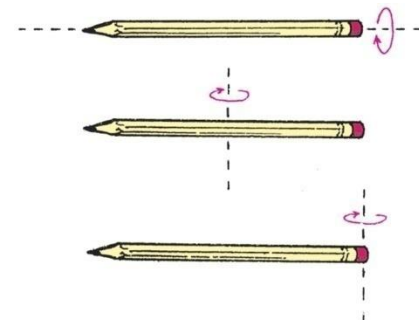
- Rotational inertia depends on the axis around which it rotates:

Eg With a pencil:

Easiest to spin here (smallest I)

Harder here

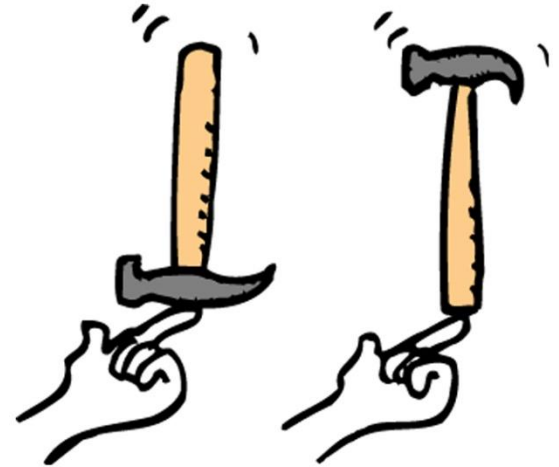
Even harder here



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Question

Consider balancing a hammer upright on the tip of your finger. Would it be easier to balance in the left-hand picture or the right-hand picture, and why?



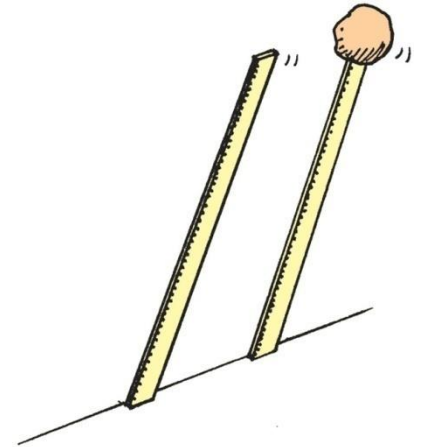
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Easier on the right, because it has more rotational inertia (heavy part further away from your finger), so is more resistant to a rotational change.

Clicker Question

Consider the meter sticks shown, one with a glob of clay at the top end. If released from upright position, which reaches the ground first?

- A) The one without the clay
- B) The one with the clay
- C) They both reach the ground together



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Answer: A

The one without the clay! Try it! Because the stick-with-clay has more rotational inertia, so resists rotation to ground more.

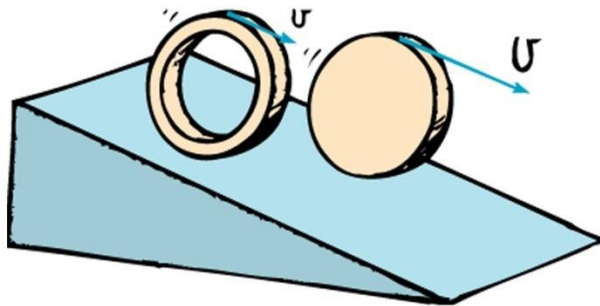
More about rotational inertia: rolling objects down a hill...

- Which rolls to the bottom of an incline first, a solid ball, a solid cylinder or a ring?

First ask: which has smallest rotational inertia? – since this will resist rolling the least, so will reach the bottom first.

The shape which has most of its mass closest to the center has least rotational inertia, i.e. first is ball, second is cylinder, and last is the ring.

(In fact this is independent of size and mass, it just depends on their shape!)



- Fig 8.14 in your text illustrates some rotational inertia values of various objects – you don't need to learn these, but do try to understand why the bigger ones are bigger from considering mass distribution.

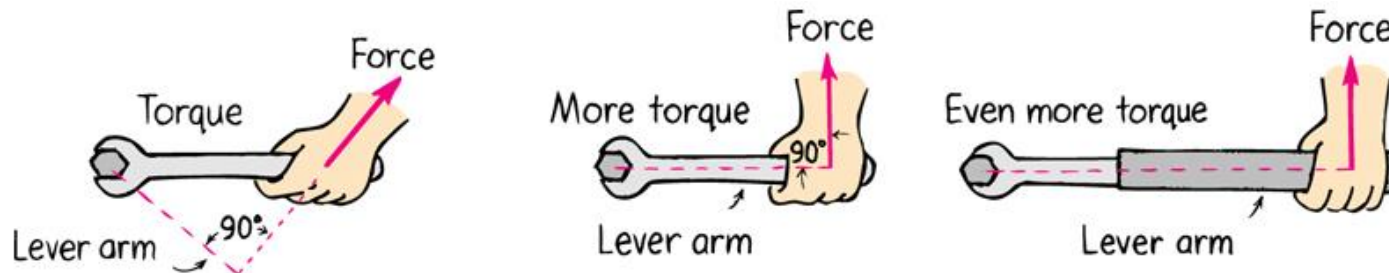
Torque

- Rotational analog of force – i.e. causes changes in rotations of objects.

$$\text{Torque} = \text{lever arm} \times \text{force}$$

lever arm = the perpendicular distance from the axis of rotation to the line along which the force acts.

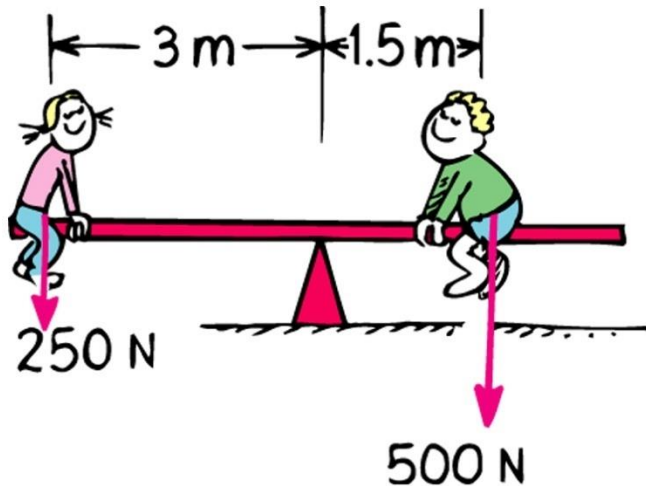
Eg. Turning a bolt



- Eg. See-saws.

The dependence of the torque on the lever arm is why kids can balance see-saws even when they have different weights – The heavier one sits closer to the center :

Larger F x Smaller lever-arm = Smaller F x Larger lever-arm.

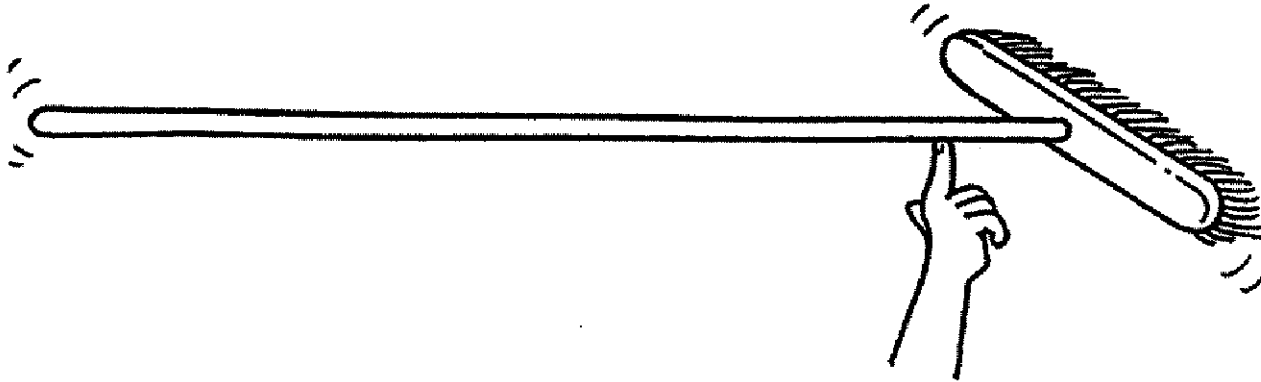


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- **Mechanical equilibrium:**

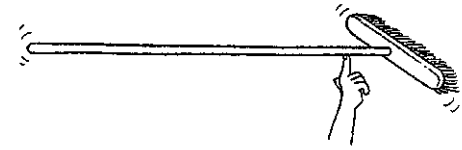
not only $\Sigma F = 0$ (chap 2) but also $\Sigma \text{torques} = \text{net torque} = 0$

Clicker Question



The broom balances at its center of gravity. If you saw the broom into two parts through the center of gravity and then weigh each part on a scale, which part will weigh more?

1. Short broom part
2. Long broom part

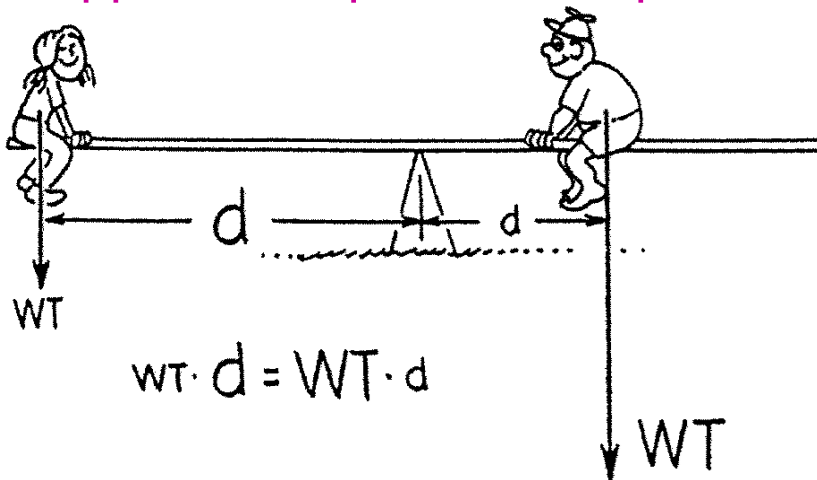


The broom balances at its center of gravity. If you saw the broom into two parts through the center of gravity and then weigh each part on a scale, which part will weigh more?

- ✓ 1. Short broom part
2. Long broom part

Answer: 1. short broom part.

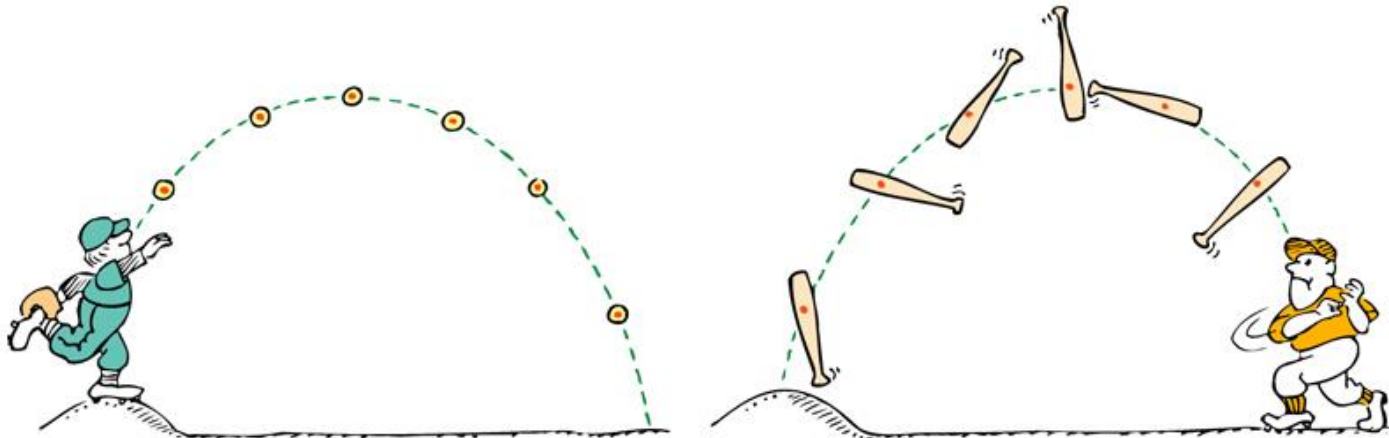
The short broom part balances the long handle just as kids of unequal weights can balance on a seesaw when the heavier kid sits closer to the fulcrum. Both the balanced broom and seesaw are evidence of equal and opposite torques—not equal weights.



Center of mass/Center of gravity

- Center of mass (CM) = average position of all the mass that makes up the object.
- Center of gravity (CG) = average position of weight distribution
 - So $CM = CG$ for objects on earth. We'll use CM and CG interchangeably.
- Often, motion of a body is complex, but CM motion is very simple:

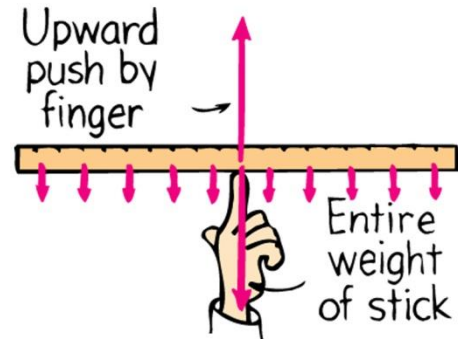
Eg. Any shaped object thrown in the air may spin in a complicated way as it falls, but the CM always follows a parabola (as if it were a point object, or ball)



Locating the CM

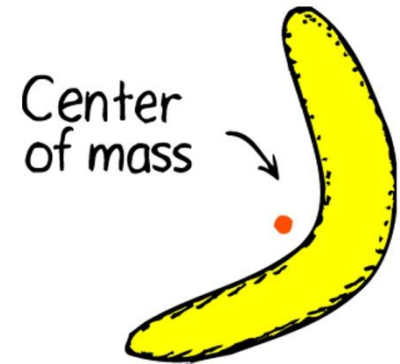
- When object is symmetric, it's simple eg. For a meter stick, CM is at the center. It acts as if all the mass is concentrated there.

All the small arrows indicate gravity along the stick – can combine to single large arrow acting downward through CM.



- Sometimes, the CM is outside of the object.

Eg. A hollow ring, CM in the center, Or banana:



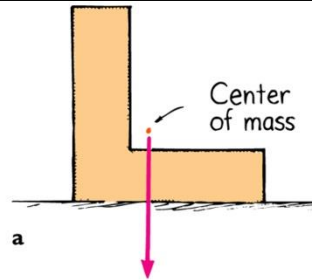
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- (Generally: if freely suspend an object from any point, the CM lies somewhere along the line vertically down from it. So, to determine exactly where, suspend it freely from some other point on the object, let it adjust, draw again the vertical line: the intersection of the two lines gives CM)

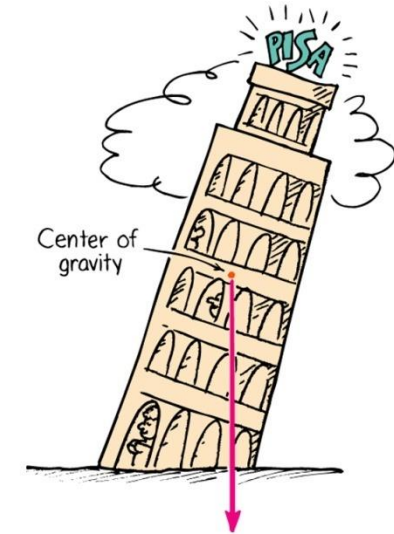
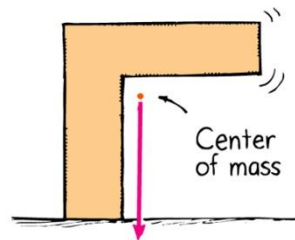
Stability

- Stable equilibrium – if vertical line down from CM falls inside the base of object.

stable



unstable



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So often design objects with a wide base and lower CM.

Clicker Question

Why should you not leave the top drawers of a heavy cabinet open while the others are closed ?

- A) The center of mass becomes too high to be stable, so it would tip
- B) The center of mass would extend in front of the cabinet, beyond the support base, so it would tip
- C) The torque exerted by the gravitational force on the drawers decreases
- D) It looks messy

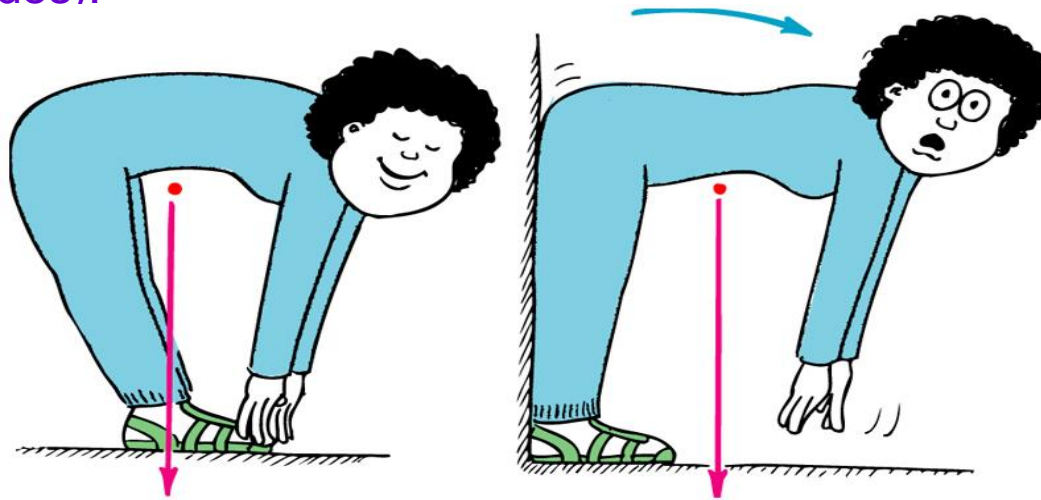
Answer B:

Objects are stable if a vertical line dropped from their COM goes through the base of the object. Leaving the top drawers open makes it look like the Γ shape on previous slide.

Example

- Why is not possible for a flexible person to bend down and touch her toes keeping legs straight, while standing with her back against a wall? (*Try this!!*)

Hint: Deduce first the CM of the person, approximating her as an L-shape (see last slides).



If she leans back, the CM is above the base (her feet) - stable

If she can't lean back, CM is no longer above her feet - unstable

The torque from gravity acting on the upper half of the body is larger in the RH case because the lever arm is longer.

Related problem: Try getting up from a chair without putting your feet under your chair.

Centripetal Force

- Is *any* force that is **directed toward a fixed center**.
- Often leads to motion in a circle – then, force is inwards, towards center of circle

Examples

Moon orbiting earth

Electrons orbiting nucleus in atom

Whirling object at end of a string

Car rounding a bend

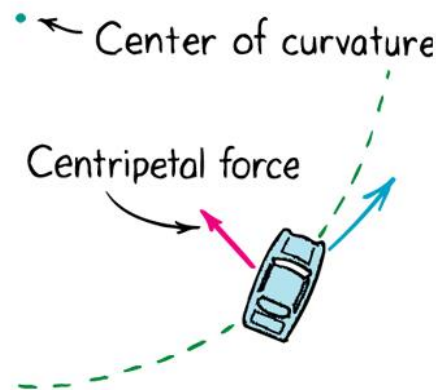
Centripetal force

Gravitational force

Electrical force

Tension in the string

Friction between tires and road



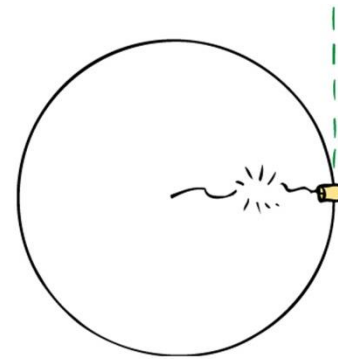
Centrifugal force

- When *you* are the object moving in a circle, you feel an **outward force** – called centrifugal force. It is a type of “inertial” force , as it is a result of rotation.

- First, consider again whirling can on end of string:

Common misconception: to say centrifugal force pulls outward on the can – wrong!

If the string breaks, the can goes off in a straight line because no force acts on it. It is the inward-directed centripetal force of the string that keeps it in a circle before string breaks.



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Then why, if we were the can whirling around in a circle, would we feel we are being pushed out, rather than the inward directed centripetal force?

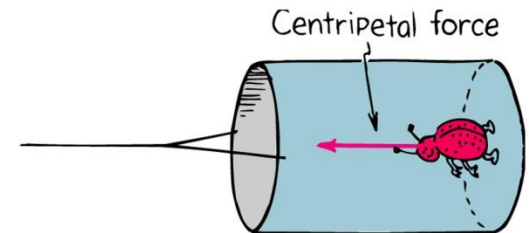
Centrifugal force continued..

...why, if we were the can, would we feel we are being pushed out rather than the inward directed centripetal force?

It's to do with the **frame of reference**: a rotating frame is a “non-inertial” frame, unlike inertial (non-accelerating) frames. **Only in inertial frames do Newton's laws strictly hold.**

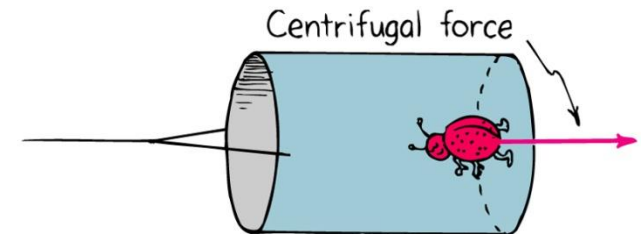
Consider a ladybug inside the can from the point of view of someone outside watching it (i.e. in an inertial frame).

Then the only force acting on it is the walls of the can on her feet, giving the inward directed centripetal force.



Now consider from the ladybug's rotating frame.

In her own frame, she is at rest. So there must be a force to cancel the wall inwardly pushing - this is the centrifugal force directed outward.



Angular momentum

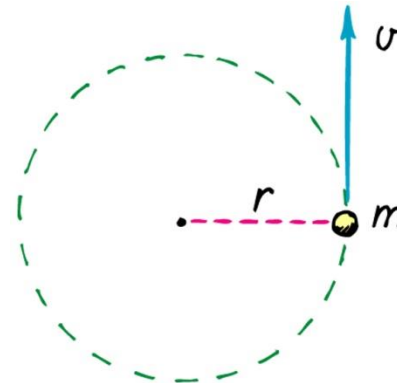
(c.f. momentum = linear momentum of Ch.6)

$$\begin{aligned}\text{Angular Momentum} &= \text{rotational inertia} \times \text{rotational velocity} \\ &= I \omega\end{aligned}$$

For an object rotating around a circular path at const speed :

$$\underline{\text{ang mom.} = m v r}$$

Eg. a whirling tin can



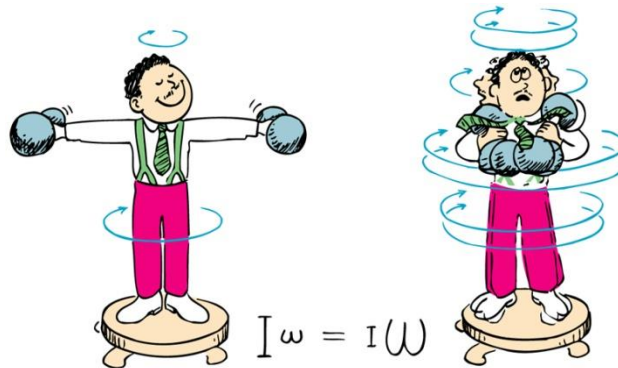
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Angular momentum is a vector quantity, but in this course, we won't deal with the (many interesting) consequences of its vector nature (eg gyroscopes).
Come ask me later if you'd like to learn more about this!

Conservation of Angular Momentum

An object or system of objects will maintain its angular momentum unless acted upon by an unbalanced external torque.

- So, if there is no net torque, ang mom is conserved.
- **DEMO:** Sit on a rotating stool, holding weights away from you. Then pull the weights in – you go much faster! Your I decreases when you pull in the masses, and your ω compensates, to keep $I\omega$ constant.



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This principle is often used by a figure skater, drawing arms and legs in to spin faster.

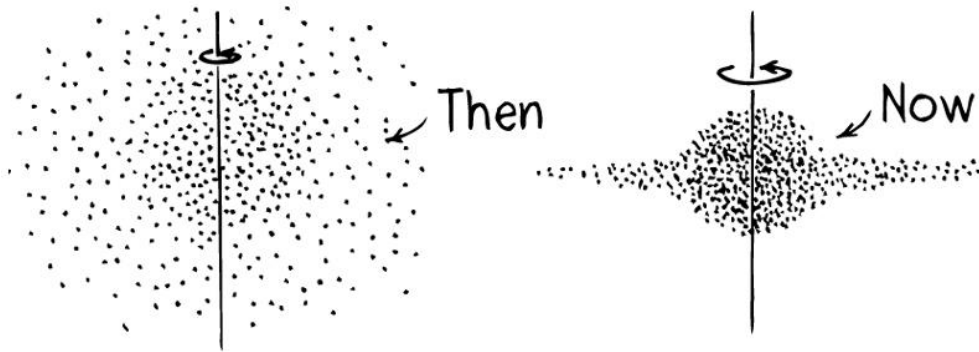
Another example: a falling cat.

Cat begins to fall upside-down but rights itself by twisting yet conserving zero angular momentum: twist parts of its body in such a way that it rotates through 180 degrees but keeping zero angular momentum!



- Another place where ang mom conservation plays big role, is in understanding motion of planets, and galaxy shapes.

e.g. we believe originally our galaxy was a huge spherical cloud of gas, rotating very slowly. Gravitational attraction between particles pulled the cloud together, so giving it a smaller radius. By ang mom cons, this means faster rotation; some stars being spun out...



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- Read in your book about how the moon is gradually getting further away from us because earth's ang mom is decreasing from ocean-water friction, and so the moon's must increase to compensate.

If all of Earth's inhabitants moved to the equator, what would happen to the length of a day?

A) The day would be longer

B) The day would be shorter

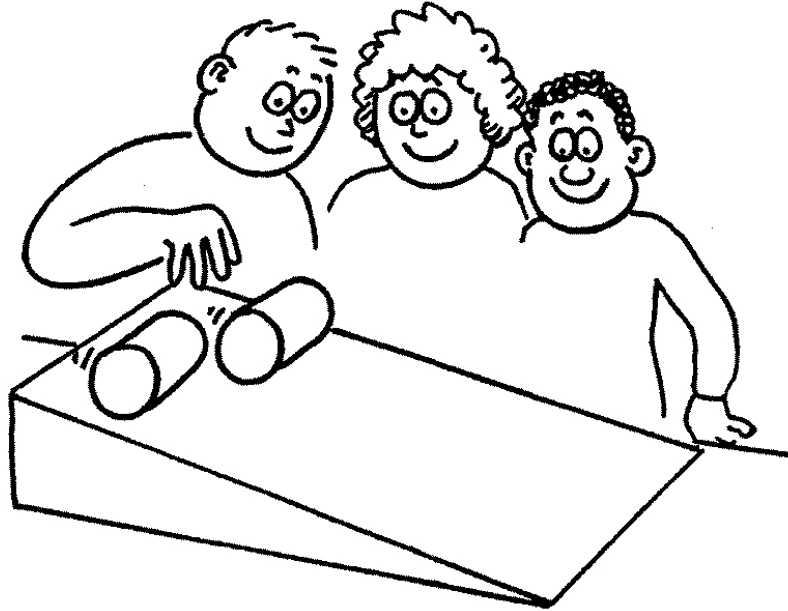
C) The length of the day would remain the same

D) There would be no day, only night

Answer:A

If mass moves to the equator, it is further from the axis of rotation, so the rotational inertia of the earth increases. Because the angular momentum of the earth is conserved, this means its rotational speed decreases, i.e. the length of the day would be longer.

Clicker Question



Which will roll down a hill faster, a can of regular fruit juice or a can of frozen fruit juice?

1. Regular fruit juice
2. Frozen fruit juice
3. Depends on the relative sizes and weights of the cans

Hint: Note that the regular juice is a liquid and so is not forced to roll with the can i.e. much of it slides. The frozen juice must roll with the can. So which has less rotational inertia?

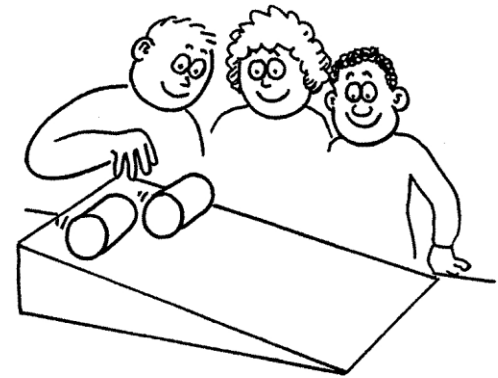
Answer: 1. Regular fruit juice

Try it at home!

The regular fruit juice has an appreciably greater acceleration down an incline than the can of frozen juice: Because the regular juice is a liquid and is not made to roll with the can, as the solid juice does. Most of the liquid effectively slides down the incline inside the rolling can.

The can of liquid therefore effectively has very little rotational inertia compared to its mass i.e the liquid doesn't rotate.

The solid juice, on the other hand, is made to rotate, giving the can more rotational inertia.



Which will roll down a hill faster, a can of regular fruit juice or a can of frozen fruit juice?

- ✓ **1. Regular fruit juice**
- 2. Frozen fruit juice
- 3. Depends on the relative sizes and weights of the cans