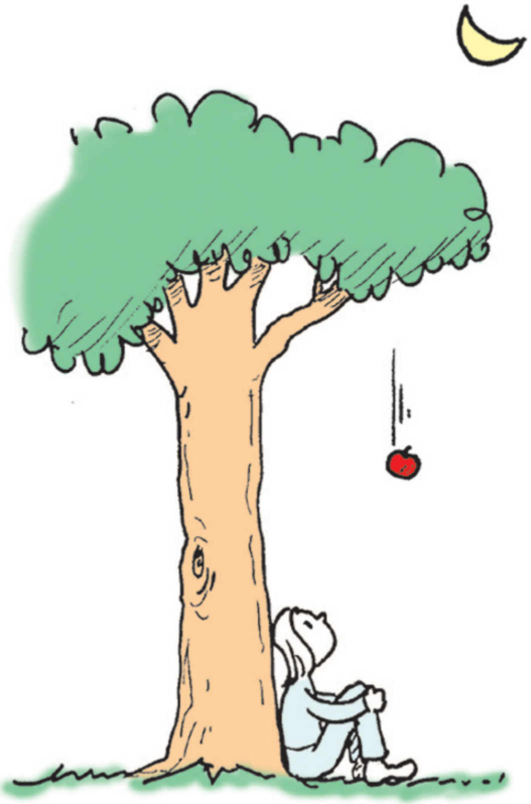
- Today: Chapter 9 (Gravity)
- Next homework posted, due Fri Oct 20

Chapter 9: Gravity

Newton: made the revolutionary connection between the circular motion of celestial bodies and the downward falling of objects on the earth:

It is the one and the same gravitational force responsible for both the apple falling from the tree and the moon orbiting around the earth!



Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

The universal law of gravity (Newton)

- Every mass m_1 attracts every other mass m_2 with a force:

$$F \sim \frac{m_1 m_2}{d^2}$$

distance between their centers

The greater (either of) the masses, the greater is the attractive force.

The closer they are to each other, the greater the force – with an *inverse-square* dependence.

- The constant of proportionality is called the **universal gravitational constant,** G = 6.67×10^{-11} N . m^2/kg^2 = 0.0000000000667 N m^2/kg^2

Tiny! So gravitational forces between everyday masses at everyday distances (eg you and me) is negligible.

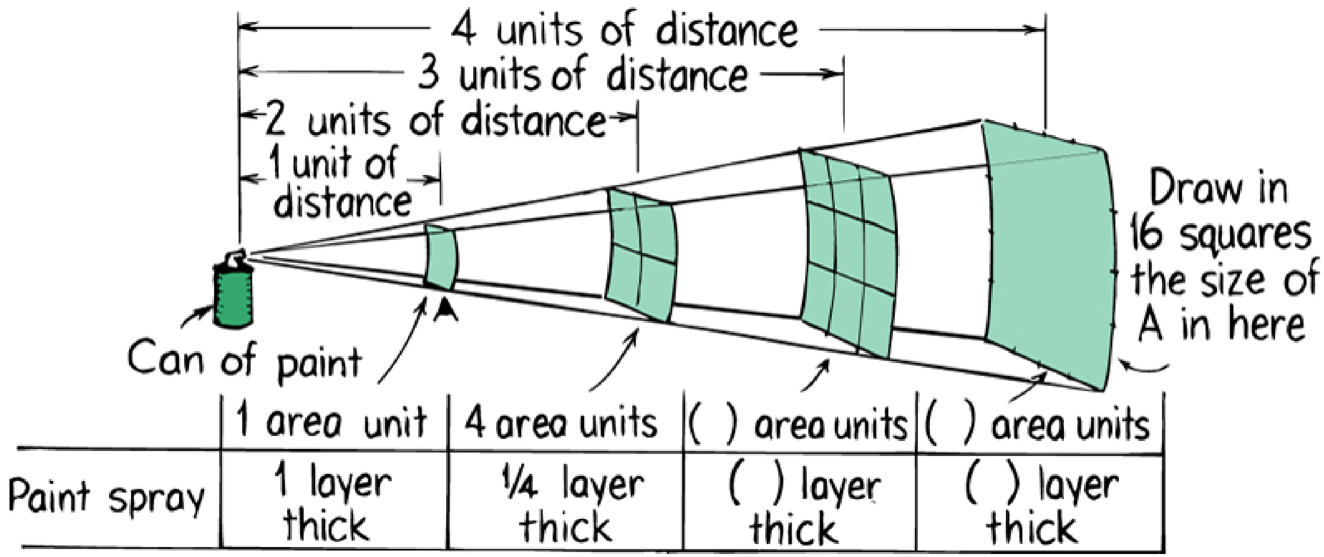
$$F = \frac{G\, m_1 m_2}{d^2}$$

Clicker Question

Distance-dependence of gravity

- Inverse-square law: $F \sim 1/d^2$

Compare with paint-spray burst out from a can: the thickness of the paint varies in the same inverse-square way i.e. if 1-layer thick at 1m, then is ¼ layers thick at 2 m etc.



Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

Distance-dependence continued…

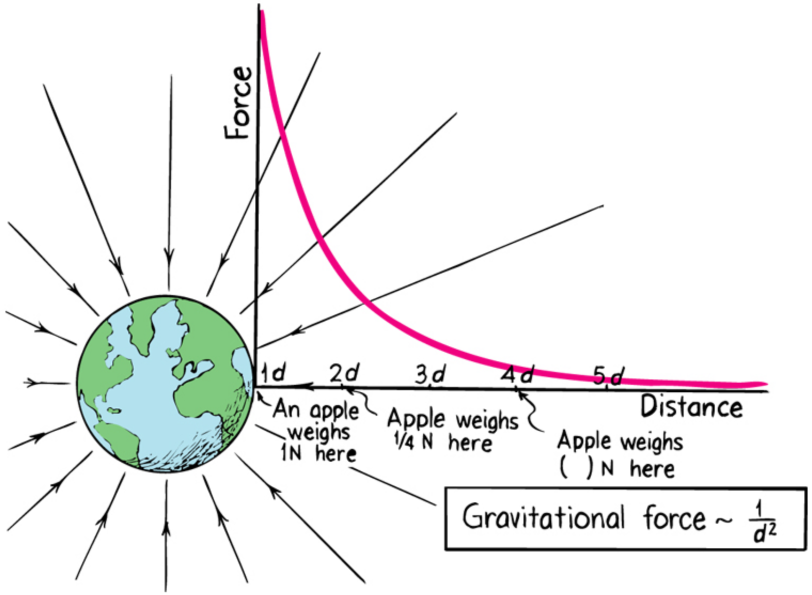
Notes (1) *d* = distance between the *center of masses* of the objects.

So when one of the objects is earth, then the relevant distance

d = radius of the earth + distance of other object from earth's surface.

6.4×10^6 m

(2) Even very very far from earth, its gravitational force is never actually zero, but it does decrease rapidly and forces from other more nearby objects would overwhelm the grav force from earth.



Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

Questions

(1) What is the force of earth's gravity on a 1-kg object at the surface of the earth? What do we commonly call this force?

$F = G\, m_{earth}\, m_{1kg}/d_{earth}^2$

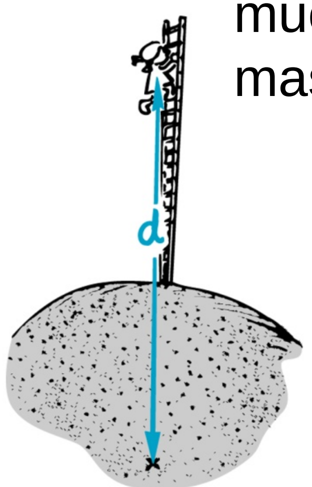
$= (6.67 \times 10^{-11})(6 \times 10^{24}\ kg)\ (1kg)/(6.4 \times 10^{6}\ m)^2 = 9.8\ N$

The force of gravity on an object is how we defined its weight.

i.e. g = 9.8 N/kg that we defined earlier, is just $g = Gm_{earth}/R_{earth}^2$.

Ordinary distances on earth are so small c.f. radius of earth, that their distance to earth's center is ~ R_{earth}, so grav force on them is just mg.

(2) If you climbed to the top of Mount Everest (height 8850 m), how much less would you weigh? Assume you eat on the way so that your mass remains fixed.



Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

At top of Everest, $d = 6.4 \times 10^6 + 8850 = 6.40885 \times 10^6$ m

So, the force is $(6.4/6.40885)^2 = 0.997$ as much

eg. If you weigh 200-lb here, then you'll weigh 199.4-lb on Mt Everest.

Clicker Question

Question:

Jupiter is about 300 times as massive as the earth but with radius about 11 as much as that of earth. On which would an apple weigh more ?

$$F = \frac{G\, m_p\, m_a}{d^2}$$

where m_p is mass of the planet and m_a is mass of the apple

So on Jupiter $F_{on\ apple} = G\ m_a(300m_E)/(11R_E)^2$ where m_E and R_E are the mass and radius of Earth

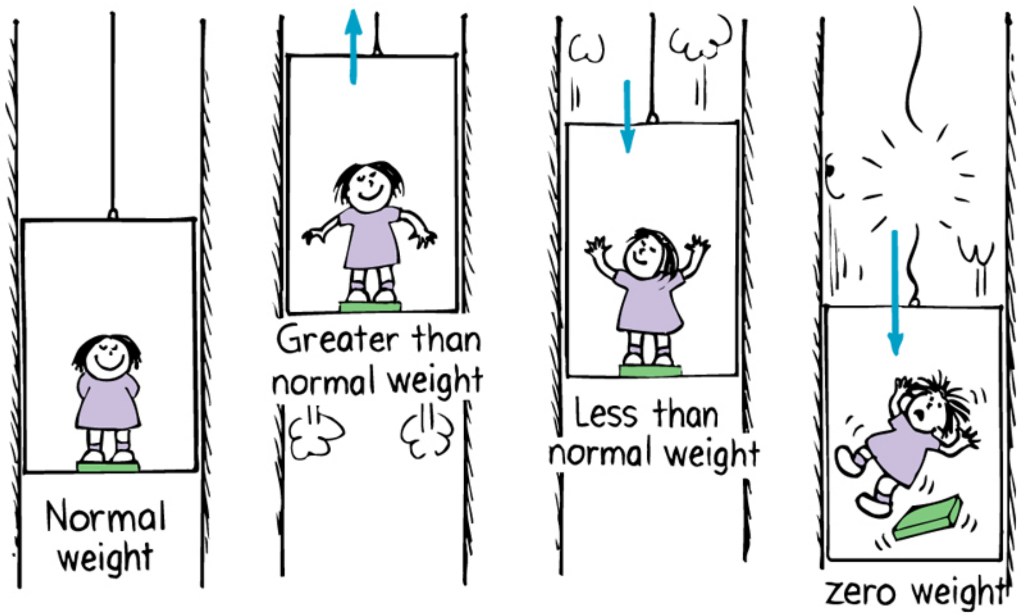
$= (300/11^2)G\ m_a m_E/R_E^2$

$= 2.6\ F_{on\ apple}$ on Earth

→ Apple weighs 2.6 times more on Jupiter than on Earth

Weight and Weightlessness

- Earlier, we defined weight as force due to gravity, *mg*.
- But if we *accelerate*, we may "feel" heavier or lighter – eg. in an elevator:



Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

Your "apparent weight" depends on your acceleration

If the elevator accelerates upwards, any scales you are standing on will read a higher weight and you feel heavier → larger "apparent weight"; if accelerates downwards, they read a lower weight and you feel lighter→ less "apparent weight".

Weight/weightless continued…

- The scales measure how much a spring inside is compressed – i.e. how much force it must exert to balance (or support) the force you are exerting on it.

- We will now **define apparent weight** to measure this instead --

Define apparent weight = force exerted against a supporting surface or a weighing scale.

(Note: your textbook calls "apparent weight" just weight at this point!)

Then, you are as heavy as you feel ! (c.f. elevator again)

Weightlessness

- If the elevator is in free fall (cable broken), then your apparent weight is zero, since there is no support force. “Weightless”.

- Gravity is still acting on you, causing downward acc. but not felt as weight.

- Same weightlessness for astronaut in orbit – he still has gravity acting on him, but since every object in his shuttle (including any bathroom scale) is falling around the earth with him, he is not supported by anything, no compression in the scales etc.

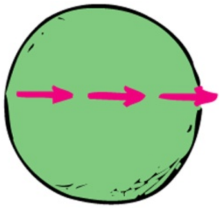
Clicker Question

Ocean Tides

- Caused by differences in the gravitational pull of the moon on the earth on opposite sides of the earth.
- Moon’s pull is stronger on the side of the earth that it is closest to; weakest on the opposite side, because F decreases with distance.

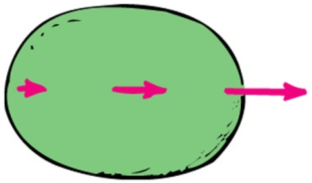
- Why does this result in *two* high-tides (and two low-tides) every day? Because when the moon is **either** closest or farthest away, you get a maximum bulge:

Imagine earth to be a ball of jello.



If moon’s force was equal at every point, then it all accelerates together towards moon.

But moon’s force is actually more like arrows here: so ball gets elongated – *both sides* effectively bulge.



Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

(moon over here somewhere)

Tides continued

So, *relative to the moon*, the tidal bulges remain fixed while Earth spins beneath – mostly it is the oceans that bulge out equally on opposite sides, on average nearly 1-m above.

Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

<u>Note: </u>the moon's pull on the earth is equal and opposite to the earth's gravitational pull on the moon. Centripetal force.

If earth was infinitely more massive than moon, moon would rotate about the earth.

Actually, they rotate about their CM which is a point inside earth, about ¾ the radius of the earth.

More on tides…

- Since earth spins once a day, any point on earth has two high tides and two low tides (on average, 1-m below average) a day.

If moon was not orbiting, then the high-low tide separation would be ¼ day, ie. 6 hours.

- But since while the earth spins, the moon moves in its orbit, it turns out the moon returns to same point in the sky every 24 hours and 50 minutes – ¼ of this is what determines the high-low-tide time difference.

- This is why high tide is not at the same time every day

- Why are there no tides in lakes?
  - Because lakes are localized; no part of the lake is a lot closer to the moon than any other part, so no big differences in moon's pull in a lake, as opposed to the oceans which span the globe…

Note also that due to the earth's tilt, the two high-tides are not equally high.

Clicker Question

<u>Question:</u> How about tides due to the sun?

The sun's gravitational force on Earth is 180 times as large as that of the moon's pull on Earth. So, what about ocean tides due to the sun?? Why are these not 180 times as strong as those due to the moon?

Because tides happen due to *<u>differences</u>* in grav pulls on one side of earth c.f. other side.

Because the sun is so far away, the $1/d^2$ factor flattens out, so the difference in its F at opposite points on the earth is very small: 0.017 %

Whereas for the moon, the difference in its grav F at opposite points on the earth is much larger: 6.7 %

Still, 180 is a big factor in the actual size of the force – and means that despite the tiny % difference, there *are* tides due to the sun, which are about half as high as those due to the moon

(180 x 0.017 % = 3 %, which is about half of 6.7 %)

Spring vs Neap tides

- Get increased (spring) or decreased (neap) tide size due to sun and moon “collaboration”:

When sun, moon are in a line with the earth, tides due to each coincide → high-tides are higher and low tides are lower than average -- **Spring** tide (nothing to do with the season).

Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

At full moon or new moon.

When lines to the moon and sun are at right angles, then high tide due to one occurs at low tide due to other → smaller than average high tides – **Neap** tide (nothing to do with your instructor)

Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

At time of half-moon.

Tides in the earth:

- Earth is molten liquid covered by a thin, solid crust → earth also experiences high and low tides! High tides are about ¼ m.
- This is why earthquakes, volcanic eruptions are more likely near a full or new moon (spring tide time).

Tides in the atmosphere:

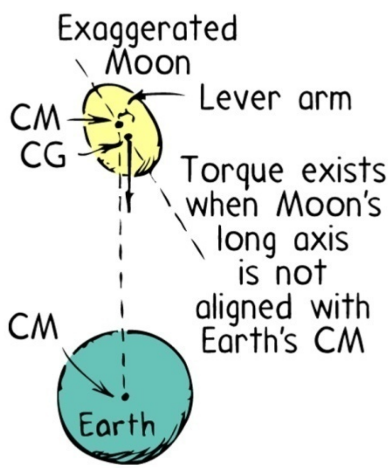
- Air also experiences tides, but we don't feel them as we are at the bottom of the atmosphere.
- Gives rise to magnetic tides in the upper atmosphere: ionosphere has many charged particles, so tidal effects lead to electric currents that change earth's magnetic field.

How about on the moon? Moon-tides

- Moon also has two tidal bulges, making it a football shape, with long axis pointed towards earth.
- But these bulges do *not* move, because the same side of the moon always faces the earth: *moon spins on its axis at the same rate at its orbital motion around earth*.

DEMO: you be the moon and try orbiting a fixed friend (earth), always keeping your face towards him/her – you find that you have to spin to do this!

- (Ages ago, it spun much faster, but then slowed down, and got locked into this synchronous orbit because of a torque action from the earth: We won't study this effect in this course, but it is interesting:

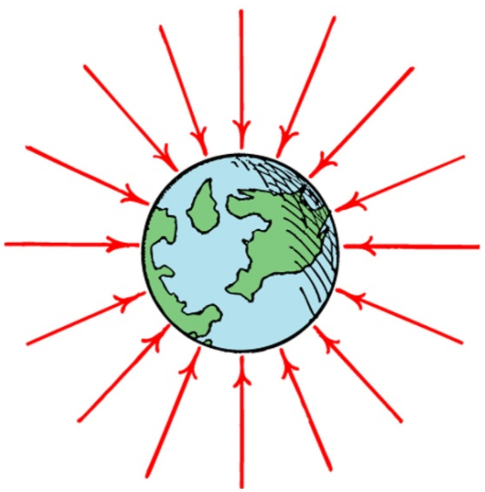


Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

As a result, on earth we only see one side of the moon.)

Gravitational Fields

- Gravitational force acts at a distance – i.e. the objects do not need to touch each other.
- We can regard them as interacting with the *gravitational field* of the other: think of this existing in the space around an object, so another object in this space feels a force towards it.


Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

Field lines have arrows indicating direction of force at that point, and are closer together when the field is strongest.

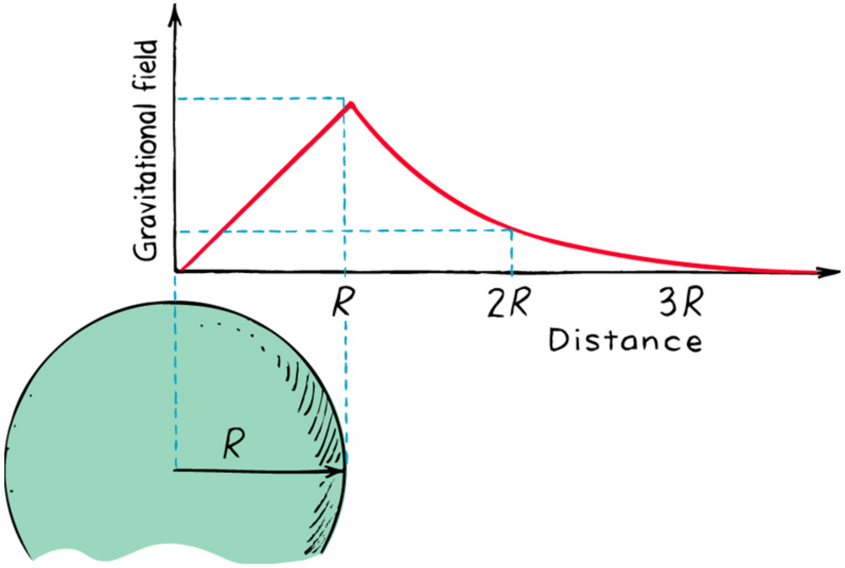
The gravitational field is a vector, same direction as the force, and strength is the force on a mass *m*, divided by that *m*:

$g = F/m$, units are N/kg

(Gravitational field inside a planet)

- We will not cover this much or examine this in this course.

- The only thing we will note is that the field increases linearly inside the planet (and falls off in the usual inverse-square way outside). It is zero right in the middle of the planet.



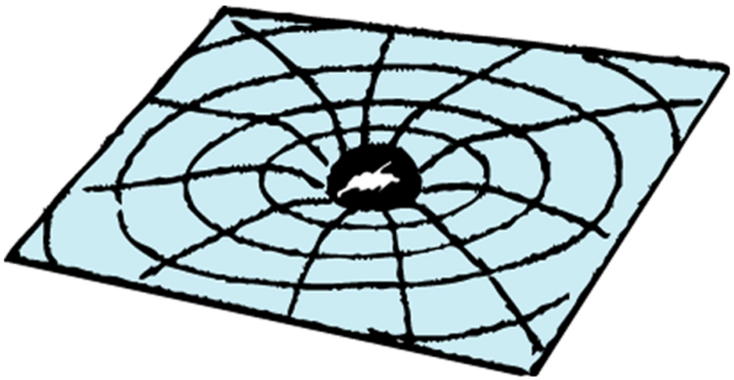
Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

- Read about it if you are interested!!

(A very little on Einstein's Theory of Gravitation)

- 1900's: Einstein's theory of general relativity involves **curved four-dimensional space-time**

Replace bodies producing gravitational fields with warped space-time.



Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

Not examinable in this course…

A little on Black Holes

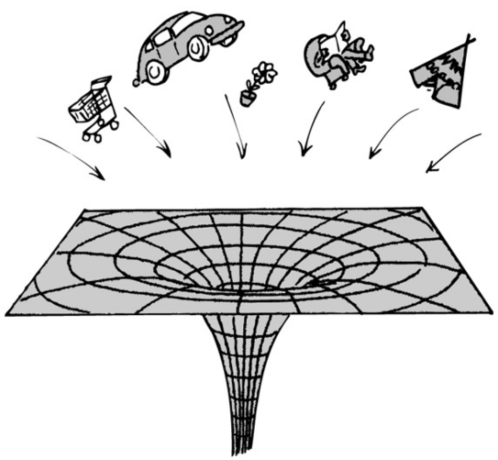
- Because grav force increases with decreasing distance, then if a massive object somehow shrinks tremendously (keeping amount of mass fixed) the grav force <u>on its surface</u> gets tremendously stronger.

- Happens for massive stars (> 1.5 of mass of our sun) when they have burnt their fuel – the stuff left condenses into an extremely dense object (neutron star) which, if large enough, continues to shrink because of its gravity.

- Consider an object on the surface of such a star – it feels increasing grav force, to the point that it can never leave it.

i.e. the speed required to overcome the grav force becomes faster than the speed of light, and *no* object can have such a speed. Called a **black hole**.

This means no object, not even light, can escape from a black hole. Anything coming near gets sucked in and destroyed (although its mass, ang mom, charge are preserved)



Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

Black holes continued…

- Since black holes are invisible, how do we know they exist?

By their grav. influence on neighboring stars – e.g. binary star systems, where have one luminous star and a black-hole orbiting each other.

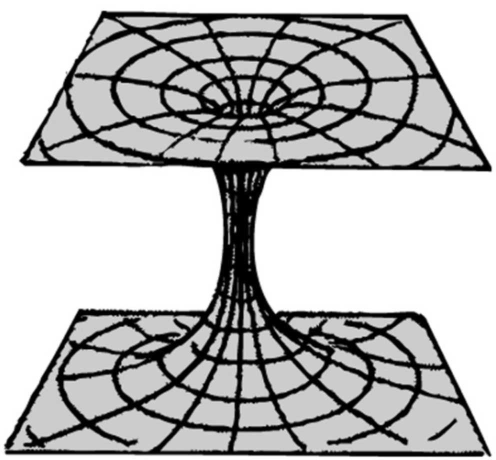
Other experimental evidence indicates massive black holes at the center of many galaxies e.g. in old ones, stars circle in a huge grav field, with an “empty-looking” center.

Galactic black holes have masses $10^6 – 10^9$ times that of our sun.

- Related, but still speculative, entity: **wormhole**

Instead of collapsing to a point, it opens out again in another part of the universe – time travel…

But still speculative (unlike black holes)



Copyright © 2006 Paul G. Hewitt, printed courtesy of Pearson Education Inc., publishing as Addison Wesley.

Clicker Question