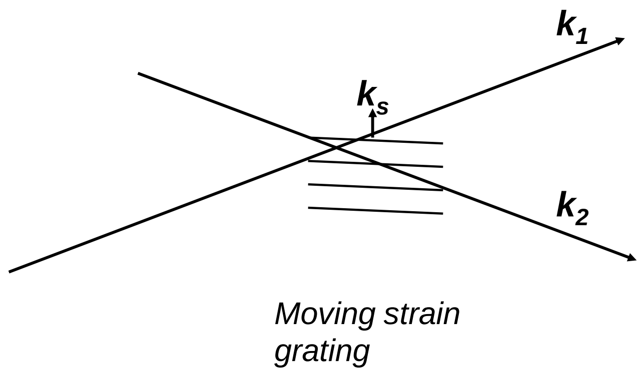
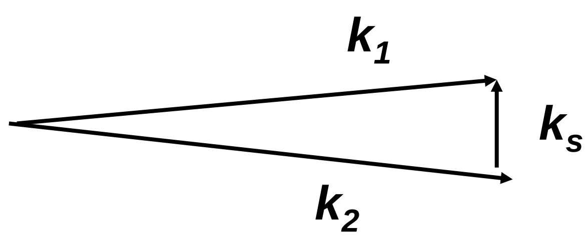
Stimulated Brillouin Scattering

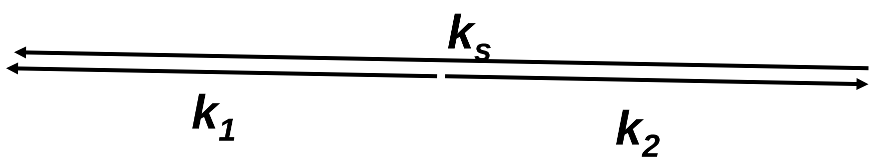
Interaction of light wave with acoustic waves

A light wave interacting with an acoustic wave through the refractive index (dielectric constant) moving grating set up by the acoustics.

$$|k_1| \neq |k_2|$$







The acoustic waves set up a displacement u in the media. The strain is the time derivative of the displacement with position.

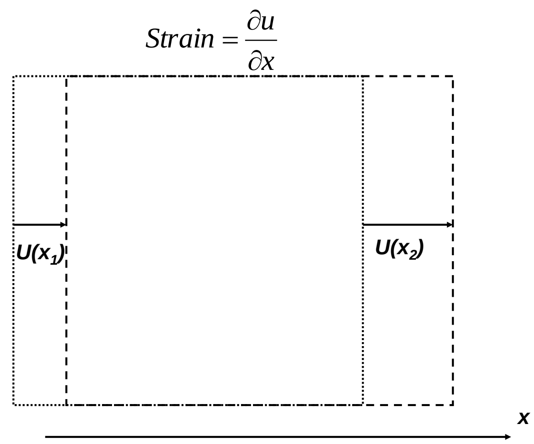
The Electromagnetic wave equation

$$\nabla^2 E_i(r,t) = \mu\varepsilon \frac{\partial^2}{\partial t^2} E_i(r,t) + \mu \frac{\partial^2}{\partial t^2}(P_{NL})_i$$

For Brillouin scattering, the nonlinear polarization source is cause by the changes in dielectric constant by the traveling acoustic waves. The acoustic waves are in turn driven by the beat waves of the electric field of light.

Acoutic waves driven by electromagnetic waves

The change in the dielectric constant caused by the strain $\frac{\partial u}{\partial x}$ is



$$\delta\varepsilon = -\gamma \frac{\partial u}{\partial x} \qquad (1)$$

where γ is a constant quantifying the changes in dielectric constant..

The changes in electrostatic energy density is given by $-\frac{1}{2}\gamma \frac{\partial u}{\partial x} E^2$. When the field is varying, the enegy density changes. The electrostrictive pressure associated with the energy change is the work divided by the strain.

$$p = -\frac{1}{2}\gamma E^2 \qquad (2)$$

Thus the force applied to a unit volume is the gradient of pressure or

$$F = \frac{\gamma}{2} \frac{\partial E^2}{\partial x} \qquad (3)$$

The equation of motion for acoustic waves driven by a force is given by

$$\rho \frac{\partial^2 u}{\partial t^2} + \eta \frac{\partial u}{\partial t} + T \frac{\partial^2 u}{\partial x^2} = \frac{\gamma}{2} \frac{\partial}{\partial x} E^2 \qquad (4)$$

Where T and ρ are the elastic constant and mass density.

The speed of acoustic waves is $v_s = \sqrt{\frac{T}{\rho}}$

How to understand equation (4)

Two electric fields and acoustic field are in the form of plane waves:

$$E_1(r,t)=\frac{1}{2}E_1(r_1)e^{j(\omega_1 t-k_1\cdot r)}+cc \qquad (5)$$

$$E_2(r,t)=\frac{1}{2}E_1(r_2)e^{j(\omega_2 t-k_2\cdot r)}+cc \qquad (6)$$

$$u(r,t)=\frac{1}{2}u_s(r_s)e^{j(\omega_s t-k_s\cdot r)}+cc \qquad (7)$$

The *r's* are the distance measured along the direction of propagation. The Laplacians are simply the second order derivatives along the *r's*.

Substituting the waveforms into (4), and neglecting the second order derivatives of the acoustic filed for slow-varying with position.

$$\left[(-j\eta\omega_s+\rho\omega_s^2)u_s-T\left(k_s^2u_s+2jk_s\frac{du_s}{dr_s}\right)\right]e^{j(\omega_s t-\vec{k}_s\cdot\vec{r})}+cc$$
$$=-\frac{\gamma}{8}\frac{\partial}{\partial r_s}\left\{E_2(r_2)E_1^*(r_1)e^{j\left[(\omega_w-\omega_1)t-(\vec{k}_2-\vec{k}_1)\cdot\vec{r}\right]}+cc\right\} \qquad (8)$$

The phase factors on two sides must be the same in order to have any meaningful effect

$$\boxed{\begin{aligned}\omega_s&=\omega_2-\omega_1\\ \vec{k}_s&=\vec{k}_2-\vec{k}_1\end{aligned}} \qquad (9)$$

With these constraints, and the assumption that $\left|\frac{\partial}{\partial r_s}(E_2E_1^*)\right|<<\left|k_sE_2E_1^*\right|$ (slow varying within one wavelength of acoustic waves, Eq (8) can be simplified as

$$2ik_sv_s^2\frac{du_s(r_s)}{dr_s}+\left(k_s^2v_s^2-\omega_s^2+\frac{j\eta\omega_s}{\rho}\right)u_s(r_s)=-\frac{j\gamma k_s}{8\rho}E_2(r_2)E_1^*(r_1) \qquad (10)$$

This relation governs the development of the acoutics waves with a source that is driven by electro-magnetic waves.

Electromagnetic wave equation driven by acoustic waves

$$\nabla^2E_i(r,t)=\mu\varepsilon\frac{\partial^2}{\partial t^2}E_i(r,t)+\mu\frac{\partial^2}{\partial t^2}(P_{NL})_i \qquad (11)$$

The nonlinear polarization caused by the acoustic wave is given by , from (1)

$$\left(P_{NL}\right)_i = \left(\delta\varepsilon\right)E = -\gamma E(r,t)\frac{\partial u(r,t)}{\partial r_i} \quad (12)$$

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From (11) and using slow-varying envelope for the electric field along the propagation direction of a plane wave

$$\left[k_1 \frac{dE_1(r_1)}{dr_1}\right]e^{j(\omega_1 t - k_1\cdot r)} + cc = j\mu\frac{\partial^2}{\partial t^2}\left(P_{NL}\right)_i \quad (13)$$

Combining (12) and (13) and by choosing terms that satisfies conservation of momentum and energy:

$$\left[k_1 \frac{dE_1(r_1)}{dr_1}\right]e^{j(\omega_1 t - k_1\cdot r)} = j\frac{\mu}{4}\frac{\partial^2}{\partial t^2}\left(-\gamma E_2 e^{j(\omega_2 t - k_2\cdot r)}\frac{\partial}{\partial r_s}\left[u_s^* e^{-j(\omega_s t - k_s\cdot r}\right]\right)_i \quad (13)$$

This equation govens the generation of E_1 by the interation of E_2 with acoustic wave u.

For $\omega_s = \omega_2 - \omega_1$, Eq (13) can be simplified as

$$k_1\frac{dE_1}{dr_1} = \frac{j\omega_1^2\gamma\mu}{4}E_2(jk_s u^* + \frac{du_s^*}{dr_s}) \quad (14)$$

For acoustic wave whose amplitude do not change quickly, the derivative with respect to r_s can be neglected,

$$\frac{dE_1}{dr_1} = \frac{-\omega_1^2\gamma\mu}{4k_1}E_2 k_s u^* \quad (15)$$