6. Optical Resonators

The microwave resonators are metal boxes or pipes

(1) to build up large field intensity with moderate input power,
(2) to act as a special and frequency filter selectively to fields,
(3) to be used in spectral analyses.

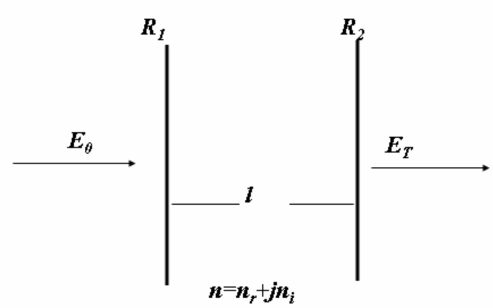
The number of modes with a frequency between ν and ν +dν per unit volume in a three dimensional resonator is

$$N = \frac{8\pi\nu^2 n^3}{c^3} d\nu \qquad (1)$$

For a box of 1cm^3, a microwave resonator contains at most one mode at a frequency of 10^9 Hz. However, at an optical frequency of 10^{14} Hz , the number of modes per unit volume can be as high as 10^9 for a bandwidth of 10^{10} Hz, unless the dimension of the resonator is reduced to the size of a wavelength. This problem can be overcome by using open resonators with a pair of mirrors with small areas. Only the mode propagating normal to the mirrors have high Q for meaningful resonance.

In a resonator of distance l between mirrors, the beam from one of the mirrors of size a_1 must be smaller than a_2 to avoid losses.

$$\frac{\lambda}{a_1} l \le a_2 \qquad or \qquad \frac{a_1 a_2}{\lambda l} \ge 1$$



Fabry-Perot resonator with plane parallel mirrors)

It can be proved, by considering multiple reflections between the mirrors, that, for an incoming electromagnetic radiation of wavelength k_0, the transmission intensify is given by

$$T = \frac{(1-R_1)(1-R_2)e^{4n_i k_0 l}}{(1-\sqrt{R_1 R_2}\, e^{2n_i k_0 l})^2 + 4\sqrt{R_1 R_2}\, e^{2n_i k_0 l} \sin^2(n_r k_0 l)} \qquad (2)$$

where the R's are the reflectance of the mirrors and the index of refraction is a complex number.

For n_i =0,

$$T = \frac{(1-R_1)(1-R_2)}{(1-\sqrt{R_1R_2})^2 + 4\sqrt{R_1R_2}\sin^2(kl)} \quad (4)$$

Case I : $R_1=R_2=R$

$$T = \frac{(1-R_1)^2}{(1-R)^2 + 4R\ \sin^2(kl)} = \frac{1}{1+\frac{4R_1}{(1-R)^2}\sin^2(kl)} = \frac{1}{1+(\frac{2F}{\pi})^2\sin^2(\frac{\pi\nu}{\nu_F})} \quad (5)$$

Where the Finesse is defined by

$$F = \frac{\pi\sqrt{R}}{(1-R)}$$

The linewidth is given by

$$\delta\nu = \frac{\nu_F}{F}$$

Meaning?

Case II *:* $R_1=R_2$ and $n_i \neq 0$.
Case III: $R_1\ exp(2n_i\ k_0 l) \approx 1$

Finesse and Q-value

Factors affecting linewidth:

- Reflectivity of mirrors
- Length of resonators
- Parallelism of mirrors
- Diffraction losses
- Imperfections in optical materials
- Coherence length

Resonator modes:

Plane-parallel mirrors of finite sizes cannot confine electromagnetic radiation. Stable confinement always involves spherical mirrors.

Hermite Gaussian mode:

From (33) of Lecture 1, the solution for propagating beam in a monogenous medium of index n are:

$$U_{l,m}(x,y) = A_1 \frac{W_0}{W(z)} H_l(\sqrt{2}\frac{x}{W(z)}) H_m(\sqrt{2}\frac{y}{W(z)}) \times$$
$$e^{[-\frac{\rho^2}{W^2(z)} - \frac{jk\rho^2}{2R(z)} - jkz + j(l+m+1)\zeta]} \qquad (7)$$

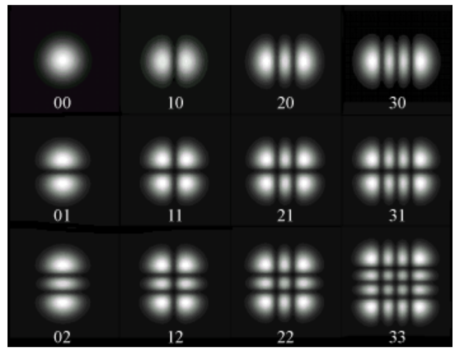
where *l,m,n* are the order of the modes, the spot size *W(z)* is

$W(z) = W_0[1 + (\frac{z}{z_0})^2]^{1/2}$

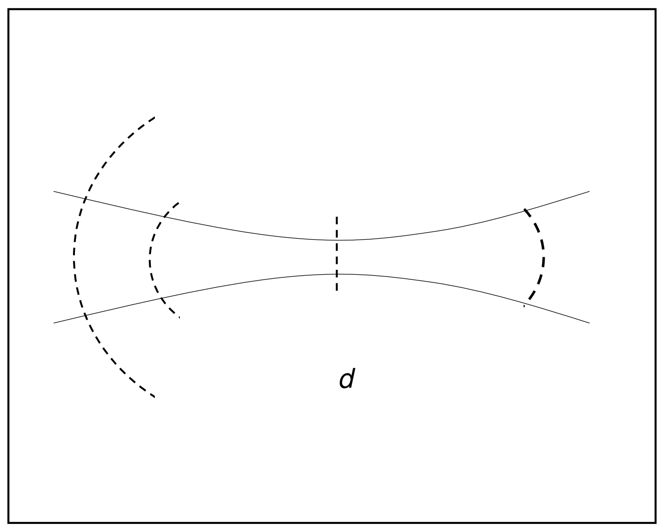
$z_0 = \frac{\pi W_0^2}{\lambda}$

The phase factor ζ is

$\zeta(z) = \tan^{-1}(\frac{z}{z_0})$.



Stable resonators can be formed by placed two mirrors at z_1 and z_2 with their radii of curvature matching the curvature of the wavefront.



Starting with a beam with a waist W_0 at $z=0$, the mirrors placed at $z=z_1$ and $z=z_2$ have radii of curvature

$$R_1 = z_1 + \frac{z_0^2}{z_1} \qquad (8)$$

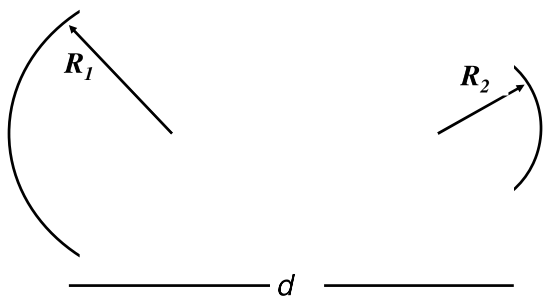
$$R_2 = z_2 + \frac{z_0^2}{z_2} \qquad (9)$$

A variety of combination of concave and convex mirrors can form a stably resonator.

Resonance frequency: p 335

Unfolding a resonator

Consider a resonator formed by two mirrors of radii of curvature R1 and R2 separated by a distance R This system can be unfolded into periodic lens sequence



Using the ABCD matrix to represent the position and slop after *m* round trips

$$\begin{bmatrix} y_m \\ \theta_m \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^m \begin{bmatrix} y_0 \\ \theta_0 \end{bmatrix} \tag{10}$$

Consider the relation between consecutive round trips

$$\begin{aligned} y_{m+1} &= Ay_m + B\theta_m \\ \theta_{m+1} &= Cy_m + D\theta_m \end{aligned} \tag{11}$$

By eliminating the angles using the relation,

$$\theta_m = \frac{y_{m+1} - Ay_m}{B}$$

$$\theta_{m+1} = \frac{y_{m+1} - Ay_{m+1}}{B}$$

We have

$$y_{m+2} = 2by_{m-1} - F^2 y_m \tag{12}$$

Where $b = \frac{A+D}{2}$ (13)

$F^2 = AD - BC = \det|M|$

The beam position parameter after m round trips can be related to the initial by

$y_m = y_{max} F^m \sin(m\varphi + \varphi_0)$ (14)

where the parameters y_{max}, and φ_0 are to be determined by the initial condition, and

$\varphi = \cos^{-1} \frac{b}{F}$ (15)

For the round trip effect is to reproduce the original position, then F=1 and

$|b| \le 1$ or

$0 \le (1 + \frac{d}{R_1})(1 + \frac{d}{R_2}) \le 1$ (16)

It is customary to express the condition for stability as

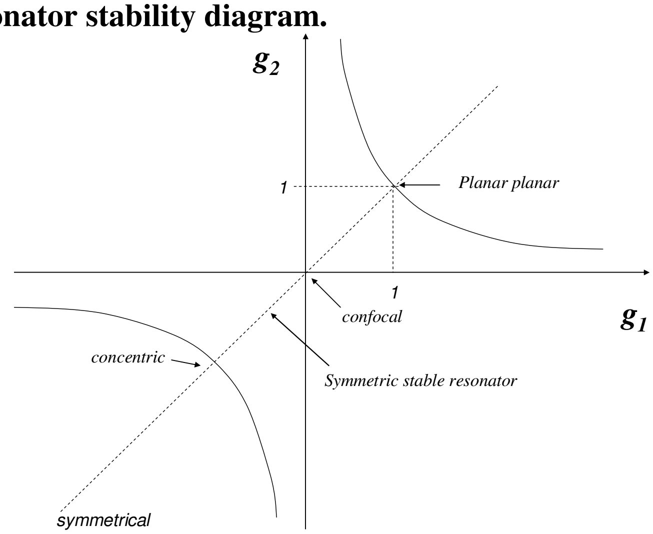
$0 \le g_1 g_2 \le 1$

where $g_1 = 1 + \frac{d}{R_1}$

$g_2 = 1 + \frac{d}{R_2}$

Concave mirrors: R<0.

Discussion of resonator stability diagram.



Discussions
Planar-planar
Planar concave
Unstable resonators
Diffraction losses due to finite mirror sizes

Homework:
1. Prove Eqs (2) and (4) and write a computer program to plot the transmitted power (E^2) as function of distance *l* for a range of variation over a few wavelengths for the case of (4) for various value of R from 0.3 to 1. You may assume that the index of refraction between the mirror is 1.

2. In a symmetric stable resonator, $R_1 = R_2$, and $g_1 = g_2$. The waist of the beam supported by the resonator is at the center. Express the beam waist using the wavelength, *g*, and the distance between the mirrors.

3. Find the ratio of the beam sizes at the waist and at mirrors in a symmetric confocal resonator.

4 . If two mirrors of radius of curvature of 10 cm and 20 cm, respectively, are used to construct a stable resonator, find the maximum distance between two mirrors.