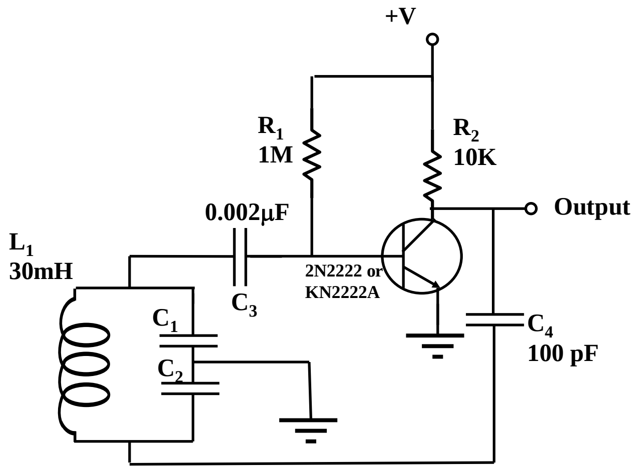
8 Laser (oscillators)

Chapter 14

An analogy: an electronic oscillator

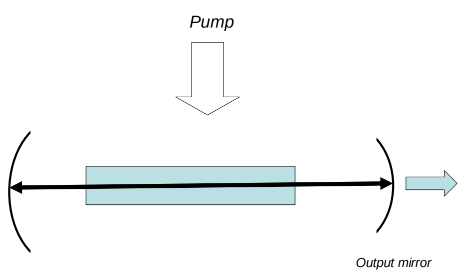


Condition for laser oscillations:

- Amplification > loss
- Total phase shift in a round trip $=2N\pi$

An oscillator comprises of

- An amplifier
- A feedback mechanism
- A frequency selection mechanism
- An output coupling scheme



Theory of laser oscillation:

(A) Amplification: discussed in Lecture 7.

(B) Optical resonator:

There are various loss mechanisms in a resonator: mirror loss, distributed losses due to scattering and unintentional absorption, etc. These losses, either distributed or localized, can be represented using loss coefficients as if they were all distributed in the entire resonator..

$$\alpha_r = \alpha_s + \alpha_{m1} + \alpha_{m2}$$

$$\alpha_{m1} = \frac{1}{2d}\ln\frac{1}{R_1} \quad . \qquad (1)$$

$$\alpha_{m2} = \frac{1}{2d}\ln\frac{1}{R_2}$$

The total mirror loss of the resonator is

$$\alpha_m = \frac{1}{2d}\ln\frac{1}{R_1 R_2} \qquad (2)$$

Define the photon lifetime in the resonator as

$$\tau_p = \frac{1}{\alpha_r c} \qquad (3)$$

Find the photon lifetime of a resonator of length 1 cm, with reflectivity 90% and 100%.

$$\alpha_r = 0.05 cm^{-1} \qquad \tau_p = 630 ps$$

The gain medium must be able to produce a gain of 5.1 % per 1 cm to overcome the loss.

In certain "high gain" lasers such as semiconductor lasers, the mirror reflectivities are 30% and 30%, the length is 500 microns. $\Rightarrow \alpha_r = 24 cm^{-1}$ $\tau_p = 1.3 ps$ The medium must amplify the signal by $2.6\text{x}10^{10}$ in 1 cm.

(C) Resonance frequency: standing waves of the resonator.

$$2kd + 2\varphi(\nu) = 2\pi q \qquad (4)$$

where q is an integer, d is the length of the resonator, assuming that the medium fill the resonator, and φ is the phase shift per unit length as discussed in Lecture 7. From (36) of Lecture 7,

$$\varphi(\nu) = \frac{\nu - \nu_0}{\Delta\nu}\gamma(\nu) \qquad (5)$$

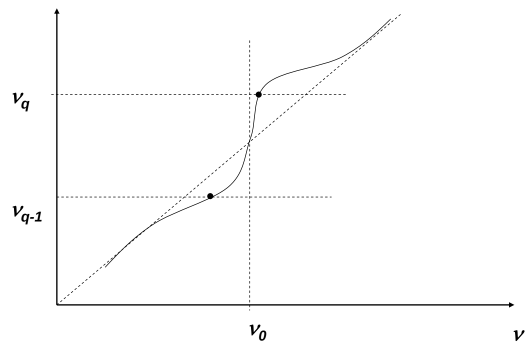
The condition of laser frequency can be obtained from (4) and (5):

$$\nu + \frac{c}{2\pi}\frac{\nu - \nu_0}{\Delta\nu}\gamma(\nu) = q(\frac{c}{2d})$$

where *q(c/2d)* is the "cold" resonance frequency of the resonator when no gain medium is present.

If the effect of the gain medium on phase shift is small, the laser frequency is simply the resonance frequency of the resonator.

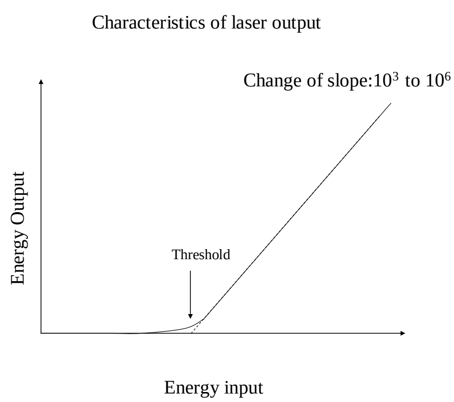
Frequency pulling toward line center



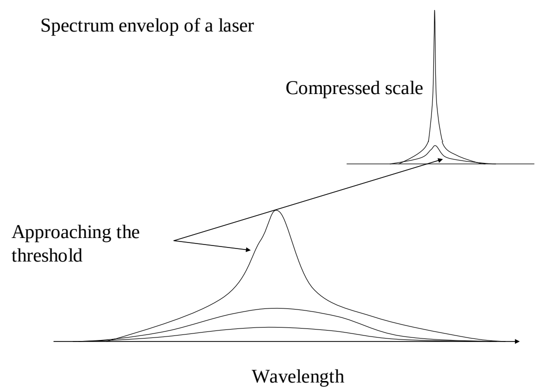
(D) Condition for laser oscillation: The "small signal gain" > loss

$$\boxed{\gamma_0(\nu) > \alpha_r} \tag{4}$$

Characteristics of laser output



The change of slope from below to above the threshold is only due to the higher directionality of the laser beam.



From Lecture 7, the gain coefficient of an amplifying medium decreases when the flux of the stimulated emission increases:

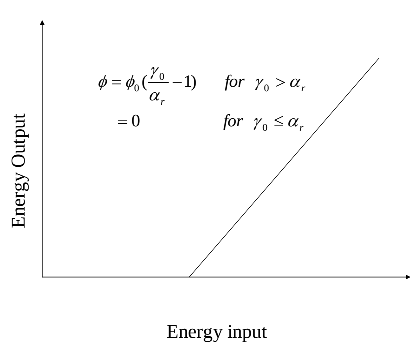
$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1+\phi/\phi_s(\nu)} \qquad (5)$$

where the gain coefficient is frequency-dependent, γ_0 is the small-signal gain coefficient at the line center, and ϕ_s is the saturation photon flux. The decrease in the small-signal gain as the stimulated emission increases is known as gain saturation. The threshold is reached when the gain equals the loss of the resonator:

$$\gamma(\nu) = \alpha_r$$

At this point, the oscillation can be sustained. The flux of the stimulated emission can be expressed as a function of the gain coefficient:

$$\begin{aligned} \phi &= \phi_s\left(\frac{\gamma_0}{\alpha_r} - 1\right) \qquad && for \;\; \gamma_0 > \alpha_r \\ &= 0 && for \;\; \gamma_0 \le \alpha_r \end{aligned} \qquad (6)$$

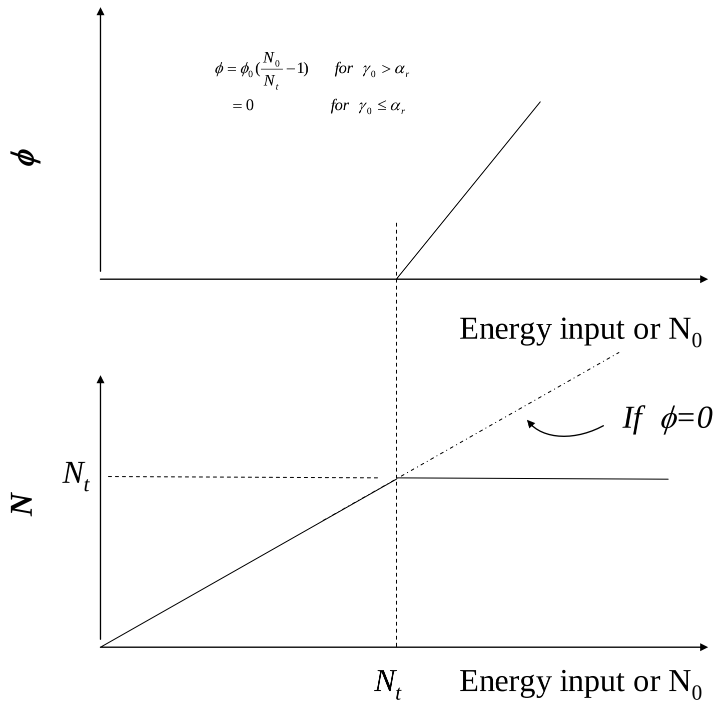


Since the gain coefficient is linearly dependent on the inversion density $N=N_2-N_1$, $\gamma_0(\nu)=N_0\sigma(\nu)$ where N_0 is the small signal inversion density. The resonator loss at the threshold can be related to the inversion density at the threshold through $\alpha_r \equiv N_t\sigma(\nu)$.

The flux of the stimulated emission can then be expressed using the inversion density by

$$\begin{aligned} \phi &= \phi_s\left(\frac{N_0}{N_t}-1\right) \qquad for \;\; \gamma_0 > \alpha_r \\ &= 0 \qquad\qquad\qquad for \;\; \gamma_0 \le \alpha_r \end{aligned} \tag{7}$$

The onset of the stimulated emission depletes the inversion density and keeps it at the same level at N_t above the threshold.regardless of the pumping rate.



Dynamics and transient effects

The dynamics of lasers is a result of the interplay between the photon number density $n=\dfrac{\phi}{c}$ and the inversion density *N*. The rate of change of photon number is governed by the rate of pumping and depletion:

$$\frac{dn}{dt} = -\frac{n}{\tau_p} + NW_i \tag{8}$$

where the photon lifetime is related to the loss of the resonator through

$\alpha_r = \dfrac{1}{c\tau_p}$ and the rate of stimulated emission is $W_i = \phi\sigma(\nu) = cn\sigma(\nu)$.

Spectral narrowing of lasing spectrum

Before studying the dynamics further, we can use this equation to explain the effect of spectral narrowing. It is useful to add an empirical term to represent the contribution of the spontaneous emission:

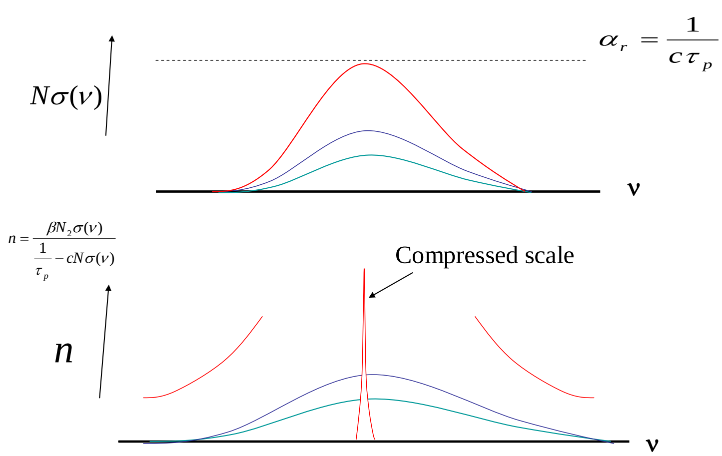
$$\frac{dn}{dt} = -\frac{n}{\tau_p} + NW_i + \beta N_2\sigma(\nu) \qquad (9)$$

where β, the spontaneous emission factor, is the fraction of the spontaneous emission that propagates along the stimulated emission.. The value of β is on the order of 10^{-3} to 10^{-6}.

At steady state, the photon density can be expressed as

$$n = \frac{\beta N_2\sigma(\nu)}{\dfrac{1}{\tau_p} - cN\sigma(\nu)} \qquad (10)$$

When N approached $N_t = \alpha_r / \sigma(\nu) = \dfrac{1}{c\tau_p}$, the threshold is reached and the photon number density can approach infinity. Thus technically, the threshold condition can never be reached. If the gain profile σ has a bell shaped distribution, the photon number density is inversely proportional to the deficit of gain from the threshold. The evolution of lasing spectrum at various levels of inversion density is illustrated in the following figure.



Dynamics of lasers

The rate equations for the photon number density, n, and inversion density, N, neglecting the spontaneous emission, are

$$\frac{dn}{dt} = -\frac{n}{\tau_p} + Ncn\sigma(\nu)$$
$$\frac{dN}{dt} = R - Ncn\sigma(\nu) - \frac{N}{\tau_s} \qquad \textbf{(11)}$$

where R is the rate of pumping to the excited state. We also assumed that, in a four level system, the lower level is nearly unpopulated and thus the decay of the inversion density due to the spontaneous emission can be approximated by the decay of the excited state only. Define a parameter $B \equiv c\sigma(\nu)$. Then (11) can be written as

$$\frac{dn}{dt} = -\frac{n}{\tau_p} + BN\sigma(\nu)$$
$$\frac{dN}{dt} = R - BNn - \frac{N}{\tau_s} \qquad \textbf{(12)}$$

Above the threshold, the steady state solutions for the inversion density and photon number density in (12) are

$$N_0 = \frac{1}{\tau_p B}, \quad n_0 = \frac{1}{N_0 B}(R - \frac{N}{\tau_s}) \qquad (13)$$

At the threshold, the photon density number is zero and the pumping rate, The pumping rate at threshold, from (13), is found to be

$$R_{th} = \frac{1}{B\tau_p\tau_s} \qquad (14)$$

The pumping rate can be normalized to the threshold pumping rate by defining a ratio $r = \frac{R}{R_{th}}$. The photon density number in (13) can expressed as

$$n_0 = \frac{1}{B\tau_s}(r-1) \qquad (18)$$

Thus the steady-state photon density number in linearly proportional to the pumping rate for normalized pumping rate *r>1*.

We are interested in the response of the photon density when a small deviation from equilibrium occurs. For small fluctuations in the photon number, , $\delta n(t)$ and inversion density, $\delta N(t)$, let

$$n(t) = n_0 + \delta n(t)$$
$$N(t) = N_0 + \delta N(t) \qquad (19)$$

It can be proved that, using the results from (13) to (18), $\delta n(t)$ and $\delta N(t)$, are coupled through the following first order differential equations:

$$\frac{d\delta n}{dt} = (\tau_p BR - \frac{1}{\tau_s})\delta N$$
$$\frac{dN(t)}{dt} = -\frac{\delta n}{\tau_p} - RB\tau_p \delta N \qquad (20)$$

These two equations lead to the equation for damped harmonic oscillator.

$$\frac{d^2\delta n}{dt^2} + \frac{r}{\tau_s}\frac{d\delta n}{dt} + \frac{(r-1)}{\tau_p \tau_s}\delta n = 0 \qquad (21)$$

Thus small fluctuations of the photon density number undergo damped oscillations, called the relaxation oscillation. The frequency of the oscillator is given by

$$\omega = \sqrt{\frac{1}{\tau_s \tau_p}(r-1) - (\frac{r}{2\tau_s})^2} \approx \sqrt{\frac{1}{\tau_p \tau_s}(r-1)} \qquad (22)$$

For lasers whose excited state lifetime is much longer than the photon lifetime, the second term in the root can be neglected to arrive at the approximation in (22).

Comparison of oscillation frequencies in various type of lasers:

Comparison of relaxation oscillation frequencies

$$\omega_r \approx \sqrt{\frac{1}{\tau_p \tau_s}(r-1)}$$

Solid state lasers

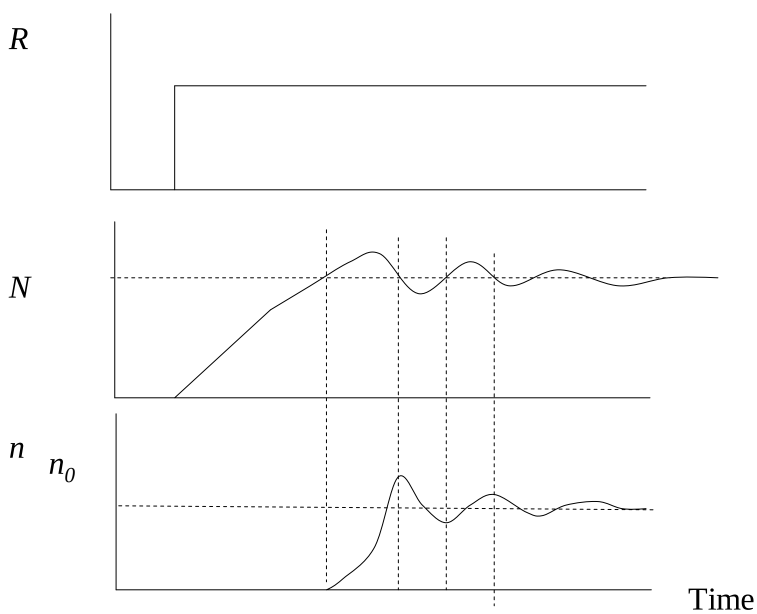
τ_p : $100ns$, τ_s : $100\mu s$ $\omega_r = \sqrt{r-1} \times MHz$

Semiconductor lasers

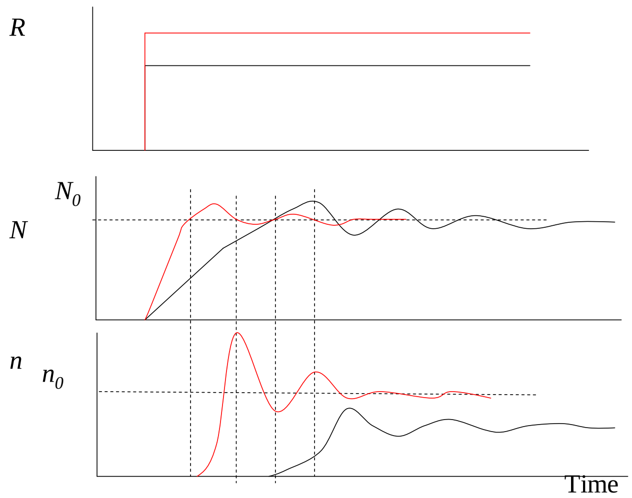
τ_p : $10ps$, τ_s : $1\ ns$ $\omega_r = \sqrt{r-1} \times 10GHz$

How about large amplitude fluctuations?

Large amplitude fluctuations

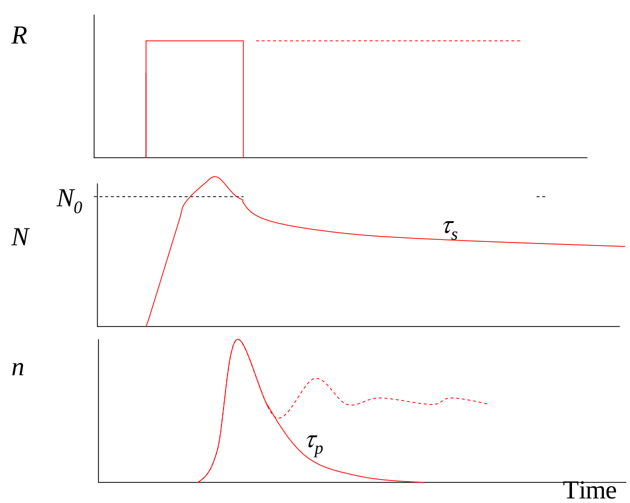


Large amplitude fluctuations

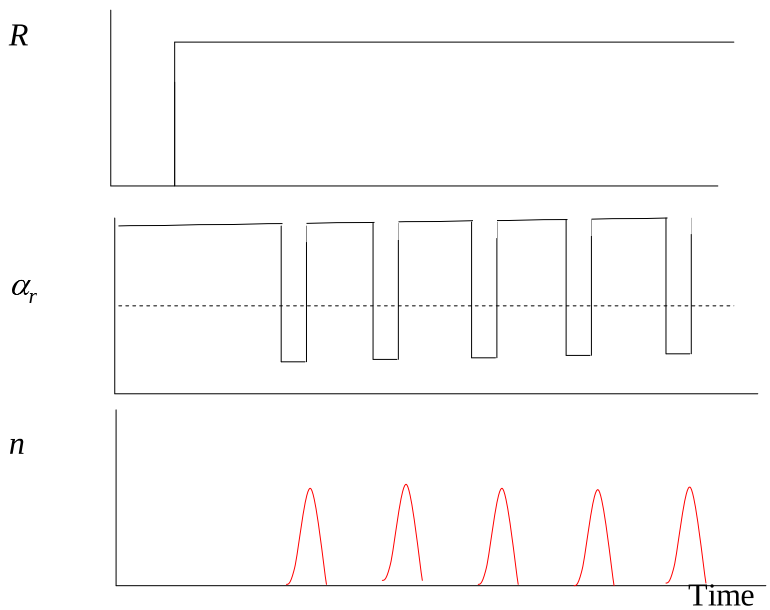


Gain switching

Gain switching

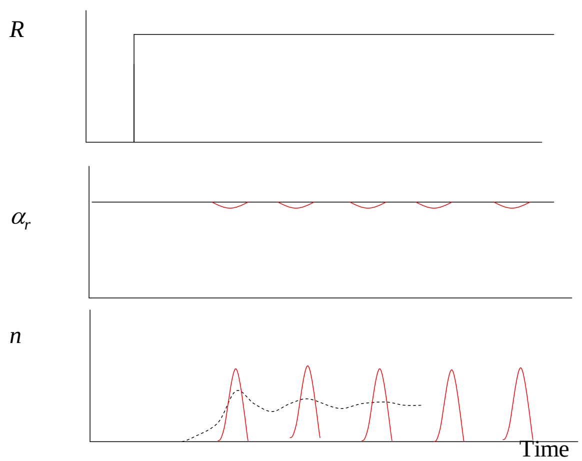


Q-switching



Q-switching

Sustained pulsation



Modulation of lasers

Consider a laser driven by a small sinusoidal signal. The response of the photon density number follows the following equation for forced oscillators:

$$\frac{d^2\delta n}{dt^2} + \frac{r}{\tau_s}\frac{d\delta n}{dt} + \frac{(r-1)}{\tau_p \tau_s}\delta n = s_0 e^{i\omega t}$$

arameter:

$$\frac{d^2\delta n}{dt^2} + \frac{r}{\tau_s}\frac{d\delta n}{dt} + \omega_0^2\delta n = s_0 e^{i\omega t} \tag{23}$$

If the photon number density is be expressed as $\delta n = \delta n_0 e^{j\omega t}$, then the response of the laser to the modulation is

$$\frac{\delta n_0}{s_0} = \sqrt{\frac{1}{(\omega^2 - \omega_0^2)^2 + (\frac{r\omega}{\tau_s})^2}} \tag{24}$$

The bandwidth of the modulation in response to the external signal is determined by the pumping rate and the lifetime of the excited state.

What is the implication?