Superfluidity of excitons and polaritons in novel two-dimensional nanomaterials

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OUTLINE

- INTRODUCTION
- SUPERFLUIDITY OF DILUTE TWO-COMPONENT 2D DIPOLAR A and B EXCITONS IN TMDC DOUBLE LAYER
- DIRECTIONAL SUPERFLUIDITY OF DILUTE 2D DIPOLAR EXCITONS IN PHOSPHORENE DOUBLE LAYER
- SPIN HALL EFFECT FOR POLARITONS IN A TMDC MONOLAYER EMBEDDED IN A MICROCAVITY
- CONCLUSIONS

Nobel prize in Physics in 2001

- **Eric Cornell** and **Carl Wieman**, NIST and University of Colorado, Boulder.
- **Wolfgang Ketterle**, MIT

Bose Einstein condensation

1925 →1995

"This discovery must be viewed as one of the most beautiful physics experiments of the 20th century" - Lev Pitaevskii

The discovery of Bose-Einstein condensation (BEC) in 1995 in dilute, ultracold trapped atomic gases is one of the most exciting developments in recent physics research.

In 2002, there are over 40 labs around the world which can routinely produce these atomic condensates.

Bose-Einstein condensation (ideal Bose gas)

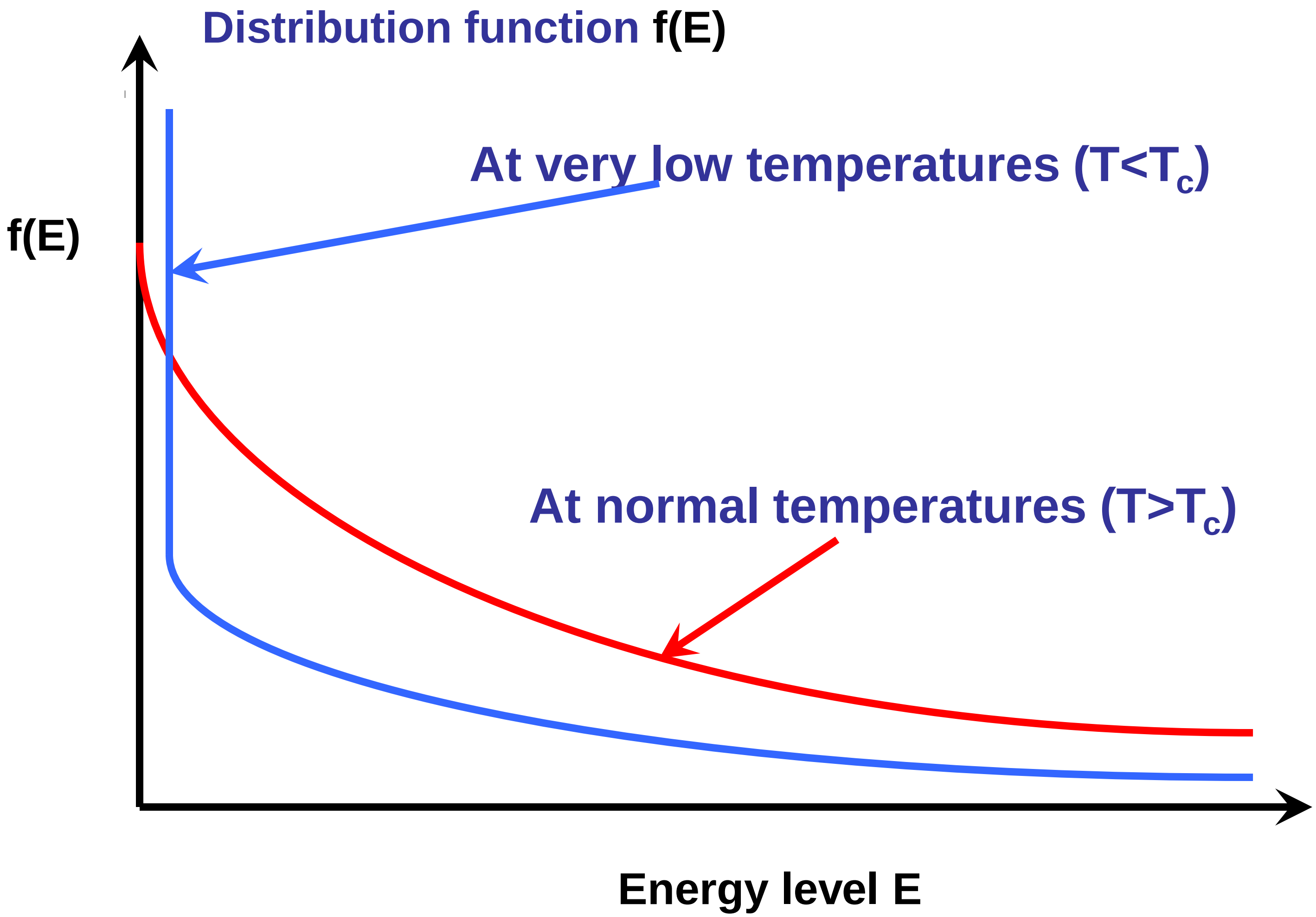
Bose, 1924 → photons (mass $m = 0$)

Chemical potential $\mu = 0$

Einstein, 1925 → mass $m > 0$

BEC: Chemical potential $\mu = 0$ at $T \leq T_c$

Chemical potential $\mu < 0$ at $T > T_c$



The Brief History of Bose-Einstein Condensation

2. July 1924: A. Einstein translated a paper of S.N. Bose which contained a new derivation of Planck's radiation law based on a statistical treatment of light quanta

10. July 1924 / 8. Jan 1925: Einstein presented a similar treatment for an ideal gas of indistinguishable particles at the *preussische Akademie der Wissenschaften*. He predicted a new condensation phenomenon.

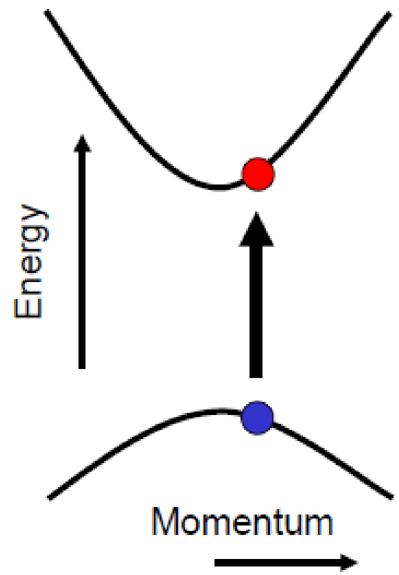
1938: Pyotr L. Kapitsa (Nobel Prize 1978) discovered the superfluidity of ^4He... the first experimental fingerprint of Bose-Einstein condensation in a dense system

5. June 1995: the advent of BEC in trapped ultracold dilute atomic gases...

^{87}Rb	5. June 1995	JILA (E. Cornell et al.)
^{7}Li	July 1995	Rice Univ. (R. Hulet et al.)
^{23}Na	Sept 1995	MIT (W. Ketterle et al.)

Making light atoms inside a solid

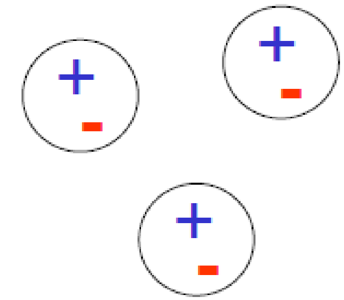
Excite electron-hole pair across a semiconductor band gap



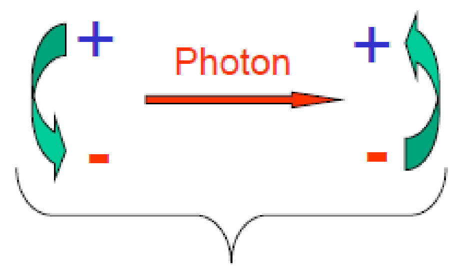
Bound by the screened Coulomb interaction to make an exciton

$$Ry^* = \frac{m^*/m}{\epsilon^2} \times \text{Rydberg}$$

$$a_0^* = \frac{\epsilon}{m^*/m} \times a_{Bohr}$$



Excitons are the solid state analogue of positronium
In GaAs
Binding energy ~ 5 meV
Bohr Radius ~ 7 nm

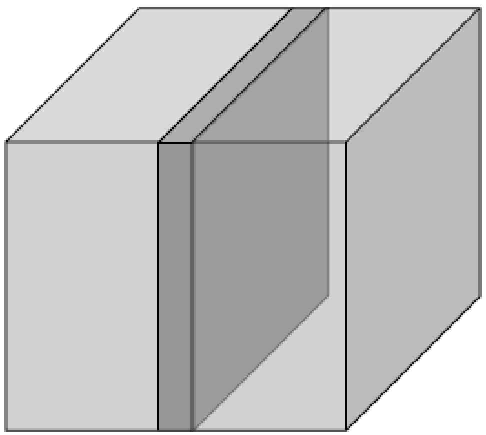


Combined coherent excitation is called a **polariton**

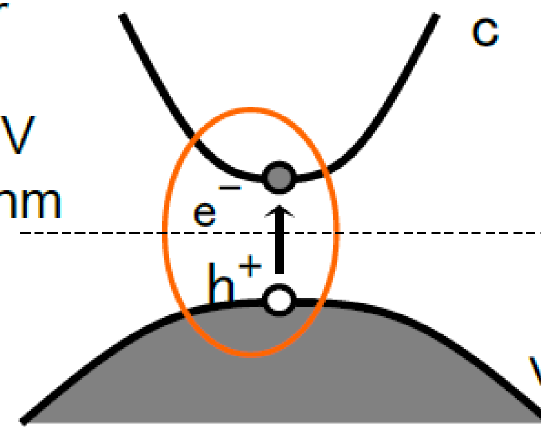
Polariton Effective Mass $m^* \sim 10^{-4}\, m_e$
$T_{BEC} \sim 1/m^*$

Quantum Well Excitons

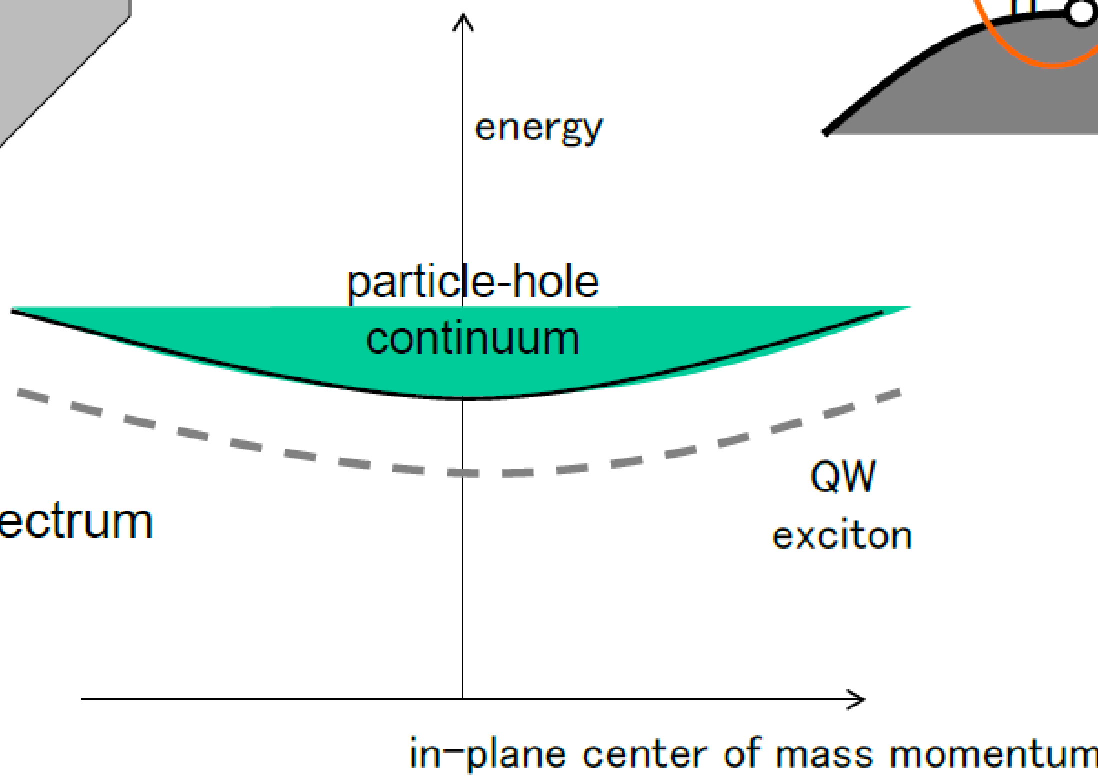
Weakly bound
electron-hole pair
EXCITON
Rydberg – few meV
Bohr Radius – few nm



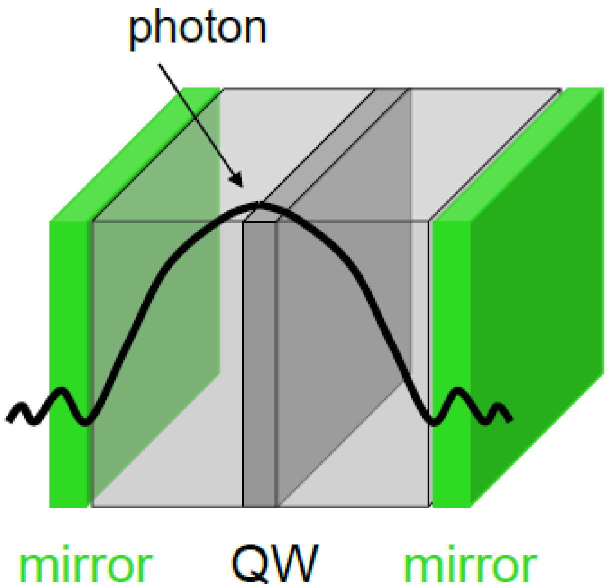
QW

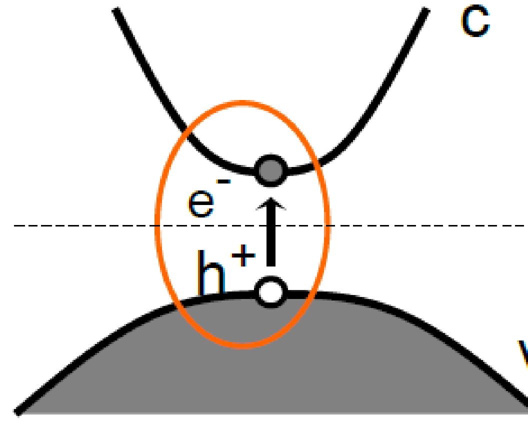


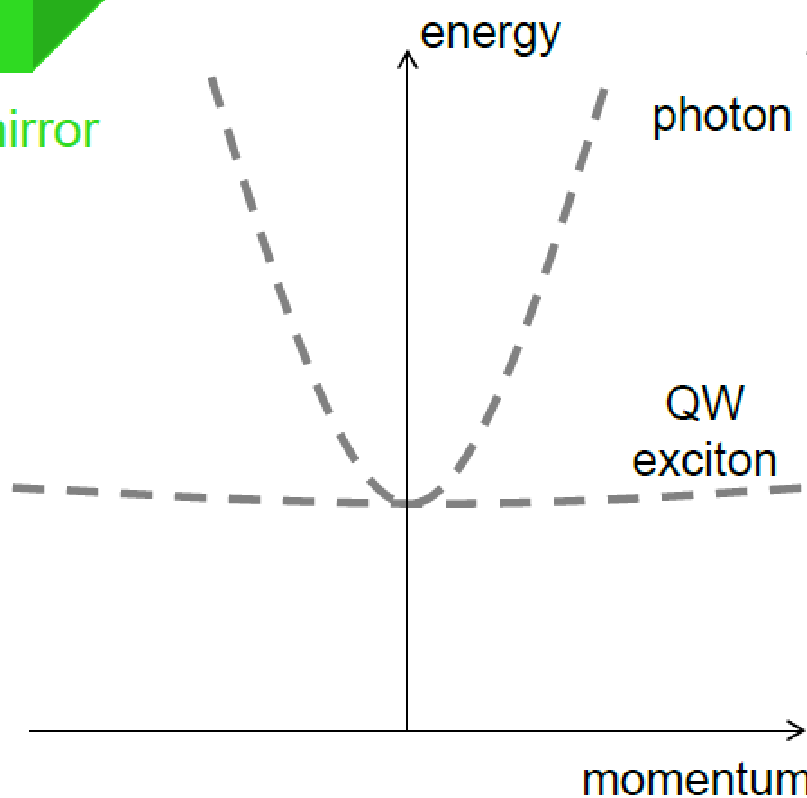
Excitation spectrum



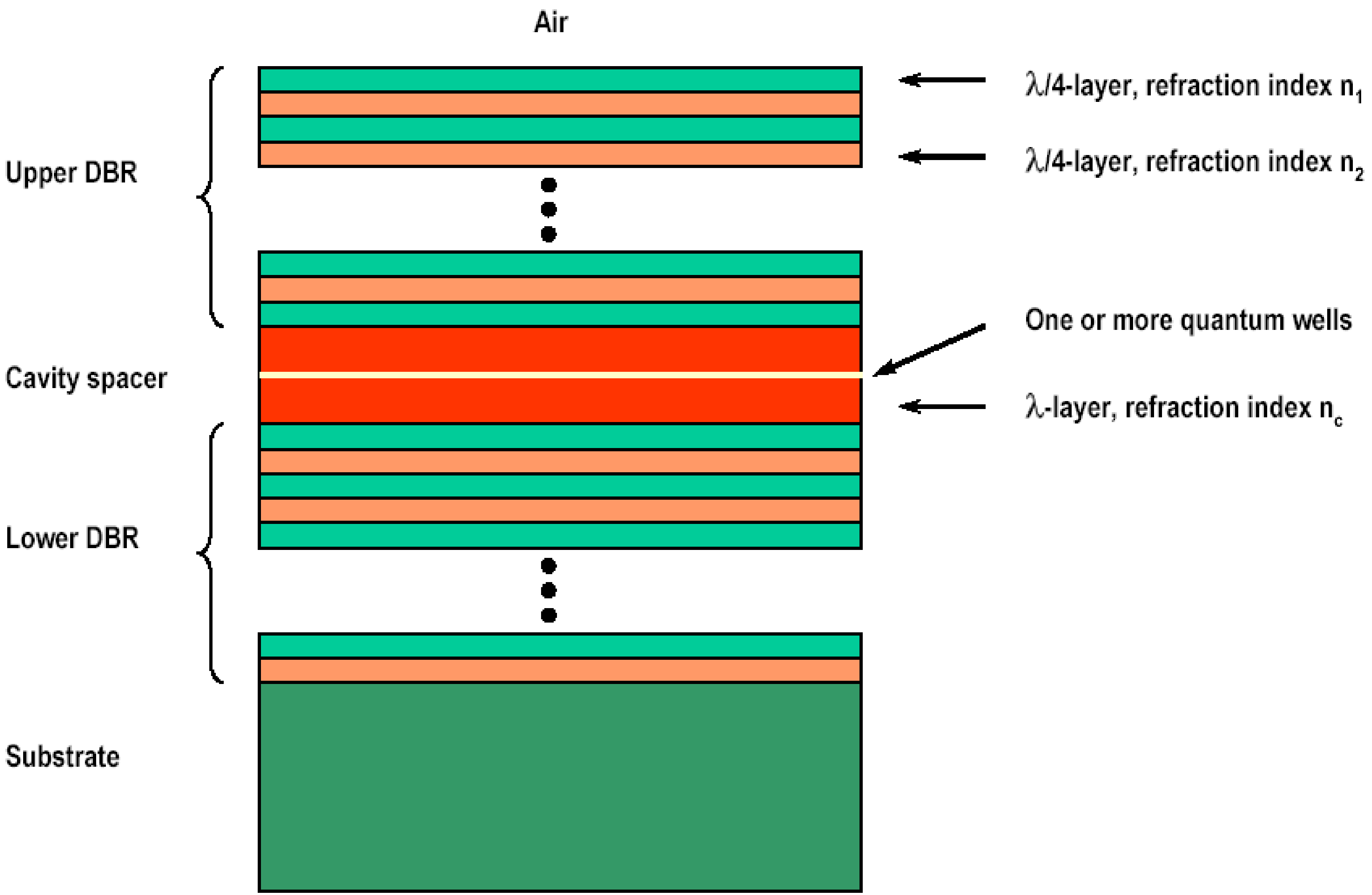
Excitons + Cavity Photons





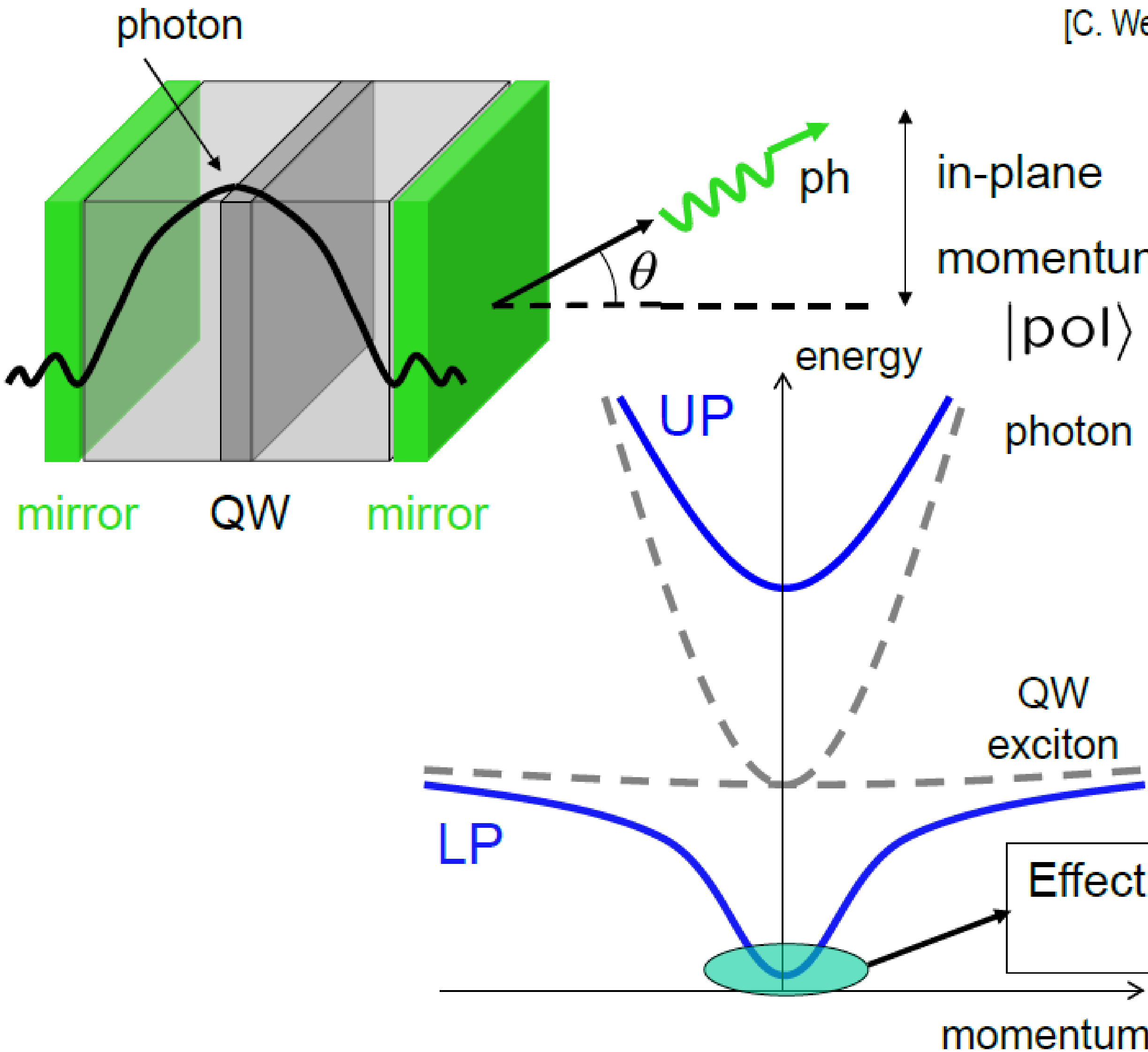


Semiconductor microcavity structure



Polaritons: Matter-Light Composite Bosons

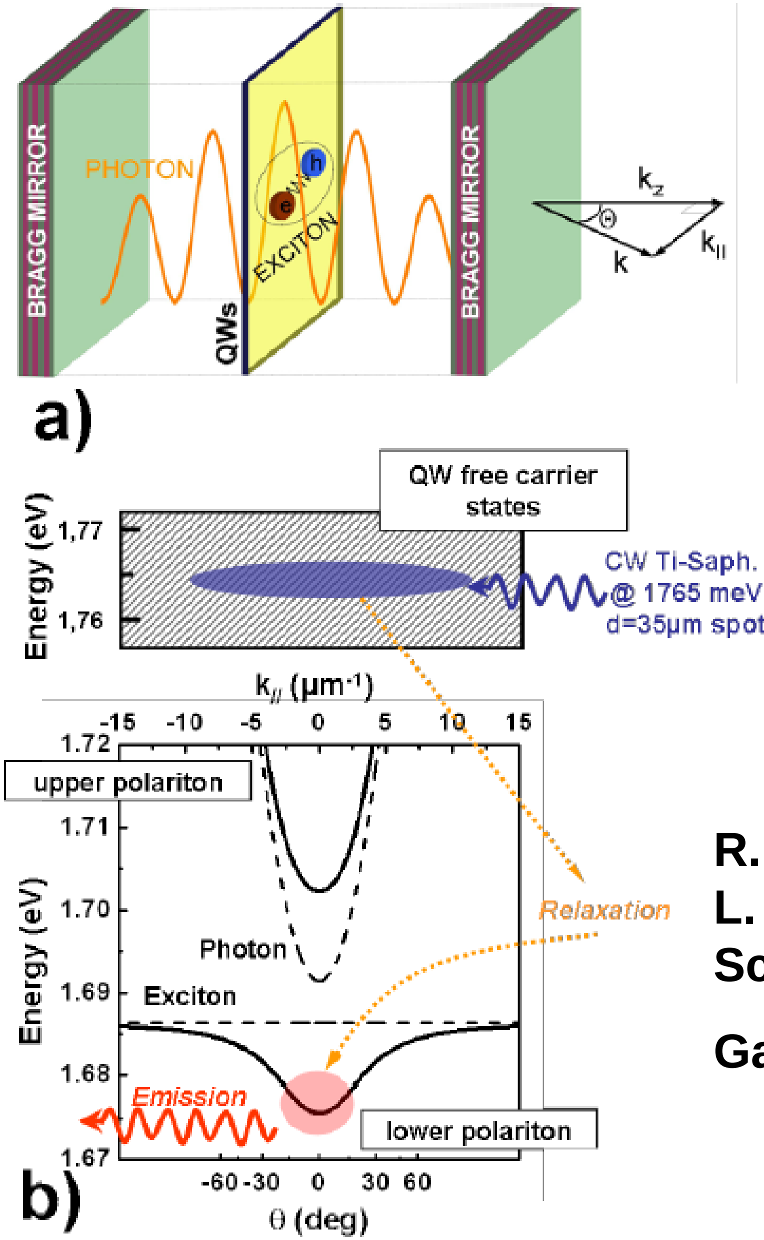
[C. Weisbuch *et al.*, *PRL* 69 3314 (1992)]



$$|\text{pol}\rangle = c_1|\text{exc}\rangle + c_2|\text{ph}\rangle$$

Effective Mass $m^* \sim 10^{-4}\, m_e$

$T_{BEC} \sim 1/m^*$



Microcavity polaritons

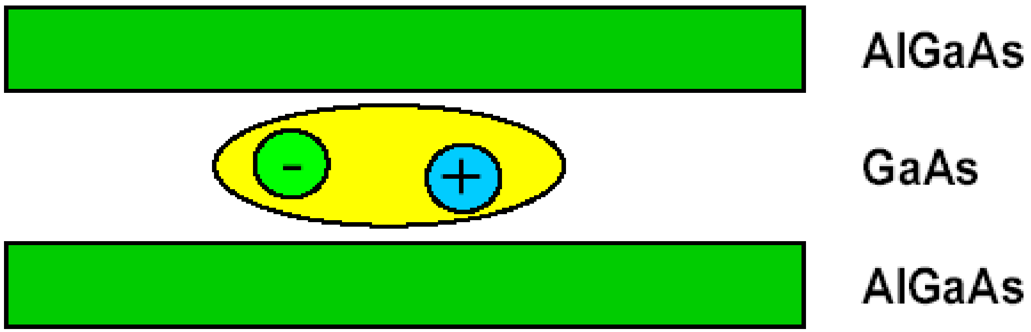
Experiments:
Kasprzak et al 2006
CdTe microcavities
Nature 443, 409 (2006).

R. B. Balili, V. Hartwell, D. W. Snoke, L. Pfeiffer and K. West, Science 316, 1007 (2007).

GaAs microcavities

Direct excitons in quantum wells

electron from conduction band + hole from valence band = exciton



Quantum confinement: enhanced binding energy and oscillator strength

k_z is not conserved

Indirect excitons in GaAs/AlGaAs Coupled Quantum Wells (CQW)

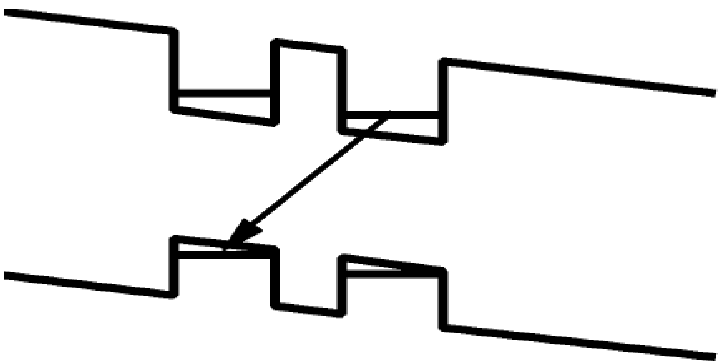
1. D. Snoke, S. Denev, Y. Liu, L. Pfeiffer, and K. West, Nature **418,** 754 (2002).

2. D. Snoke, Science **298**, 1368 (2002).

3. L. V. Butov, J. Phys.: Condens. Matter **16**, R1577 (2004).

4. J. P. Eisenstein and A. H. MacDonald, Nature **432**, 691 (2004).

5. M. Alloing et al, Europhys. Lett. **107**, 10012 (2014).



Theory:

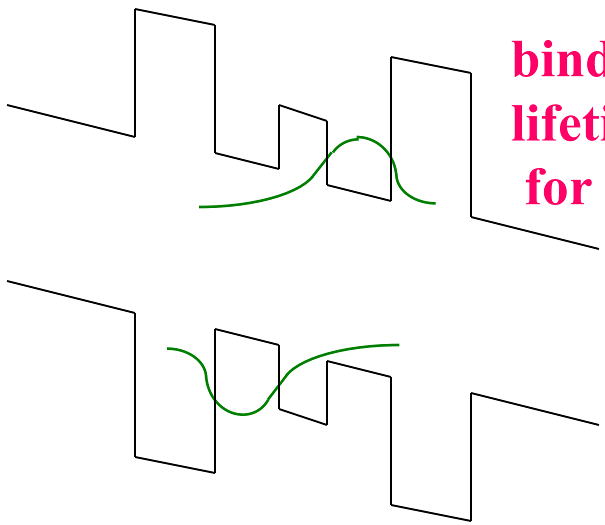
Yu. E. Lozovik and V. I. Yudson, Sov. Phys. JETP 44, 389 (1976)

X. Zhu, P. B. Littlewood, M. S. Hybertsen, and T. M. Rice, Phys. Rev. Lett. 74, 1633 (1995)

G. Vignale and A. H. MacDonald, Phys. Rev. Lett. 76, 2786 (1996)

O. L. Berman, Yu. E. Lozovik, D.W. Snoke, and R. D. Coalson, Phys. Rev. B 70, 235310 (2004).

Coupled quantum wells GaAs / AlGaAs



binding energy 4 meV
lifetime ~ 10 μs!
for D = 240 Å

spatial separation

gives long exciton

lifetime- 10 μs

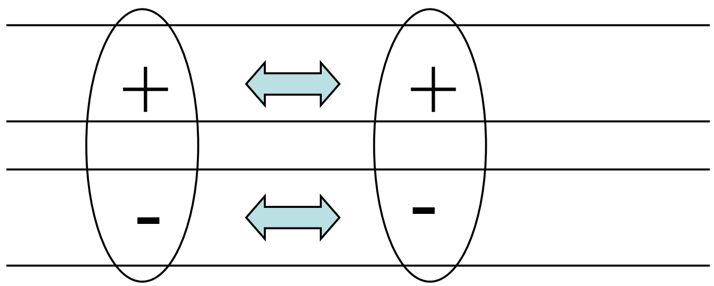
but E-field gives

tunneling current

through the structure

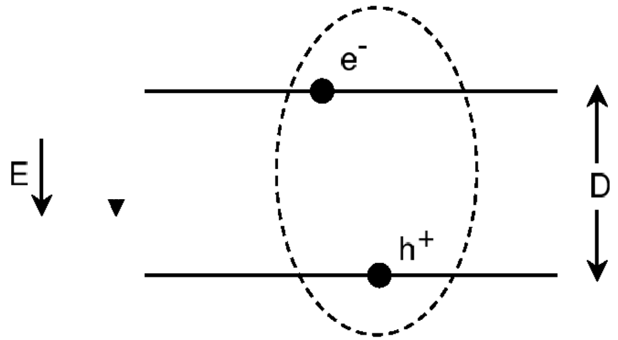
Berman and Lozovik, JETP **84**, 1027 (1997):

overall repulsive for D > 1.1a_B

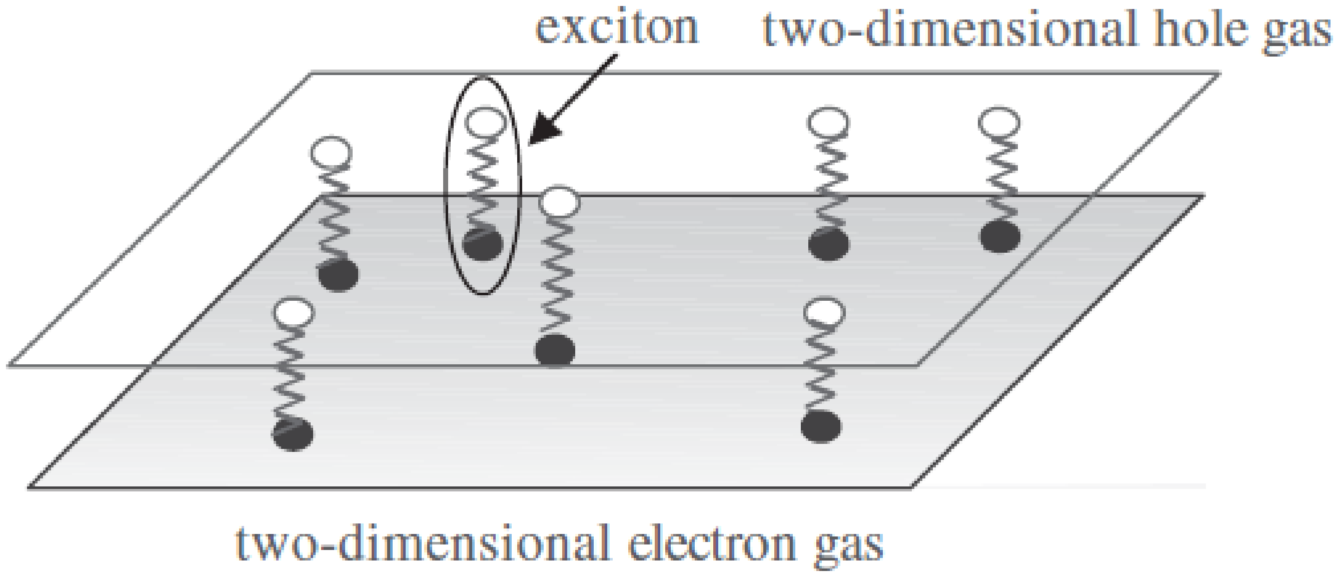


aligned dipoles
give overall repulsion

2D indirect excitons



The spatially separated electrons and holes in a double layer.



ATOMIC WORLD *versus* EXCITATIONS WORLD

PHASES

- Nonideal gas
- Effects of Bose – Einstein statistics
- Bose - Einstein condensation atoms in trap
- Superfluid helium

  Strong interaction: only 9% in Bose condensate
- Crystal phase

$T_c \approx 1nK$

•Nonideal gas of excitons

Manifestation of nonieadlity:

Blue shift of exciton line

•Manifestation of Bose induced

processes was observed

•BEC of indirect excitons

in CQW is claimed

•?

•Exciton crystal phase is claimed

$T_c \approx 4K$

Graphene

Graphene was obtained and studied experimentally for the first time in 2004 by K. S. Novoselov, S. V. Morozov, A. K. Geim,et.al. from the University of Manchester (UK).

A. K. Geim and K. S. Novoselov → Nobel Prize in Physics in 2010 for discovering graphene

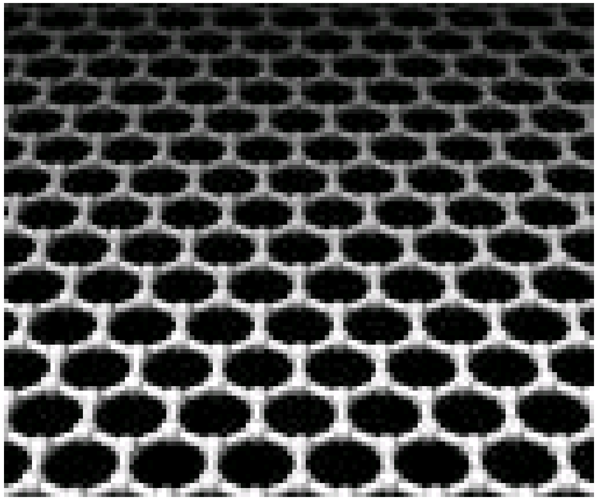
BEC and superfluidity of dipolar excitons in a graphene double layer:

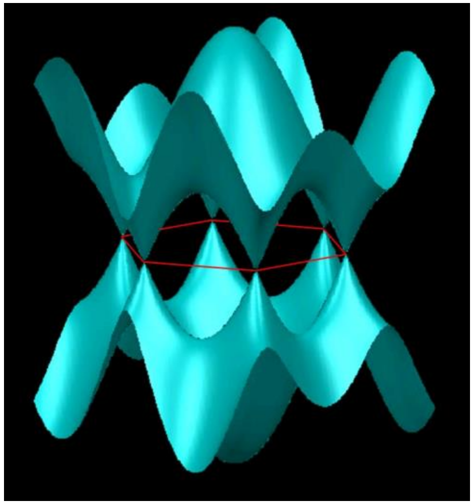
O. L. Berman, Yu. E. Lozovik, and G. Gumbs, Phys. Rev. B 77, 155433 (2008).

R. Bistritzer and A.H.MacDonald, Phys. Rev. Lett. 101, 256406 (2008).

O. L. Berman, R. Ya. Kezerashvili, and K. Ziegler, Phys. Rev. B 85, 035418 (2012).

2D atomic honeycomb crystal lattice of carbon (graphite)





Perfect Graphene crystal and resultant Band Structure.

In CQWs:

1. Fluctuations of the width of QWs

2. Gap between conduction and valence bands = 1.43 eV (GaAsAlGaAs)

In graphene:

1. No fluctuations of the width of graphene layeers (purely 2D)

2. No gap between conduction and valence bands.

Gap can be induced either by external electric or magnetic field or by impurities.

The effective mass of electrons and the energy gap in graphene equals 0

In semiconductor CQWs and graphene double layer: BEC and superfluiditry of one-component dipolar excitons

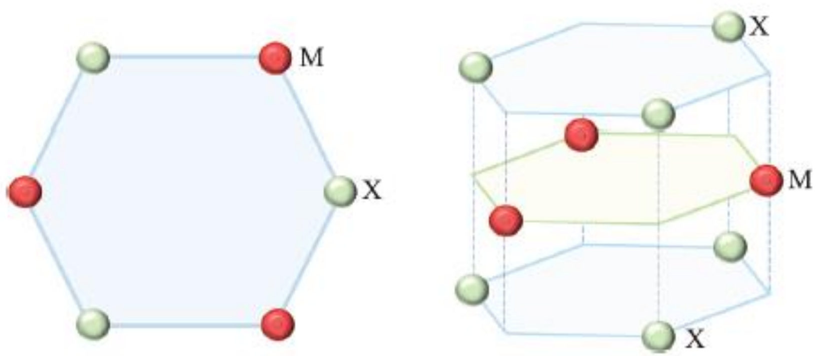
Transition metal dichalcogenide (TMDC)

Review:

A. Kormanyos, G. Burkard, M. Gmitra, J. Fabian, V. Zolyomi, N. D. Drummond, and V. Fal'ko, 2D Mater. 2, 022001 (2015)

Excitons in TMDCs: A. Chernikov, T. C. Berkelbach, H. M. Hill, A. Rigosi, Y. Li, O. B. Aslan, D. R. Reichman, M. S. Hybertsen, and T. F. Heinz, Phys. Rev. Lett. **113**, 076802 (2014).

The structure of a TMDC monolayer.



BEC of dipolar excitons in a TMDC double ilayer:

M. M. Fogler, L. V. Butov, and K. S. Novoselov, Nat. Commun. 5, 4555 (2014).

F.-C. Wu, F. Xue, and A. H. MacDonald, Phys. Rev. B 92, 165121 (2015)

BEC and superfluidity of two-component system of A and B dipolar excitons in a TMDC double layer:

O. L. Berman and R. Ya. Kezerashvili, Phys. Rev. B 93, 245410 (2016). O. L. Berman and R. Ya. Kezerashvili, Physical Review B 96, 094502 (2017).

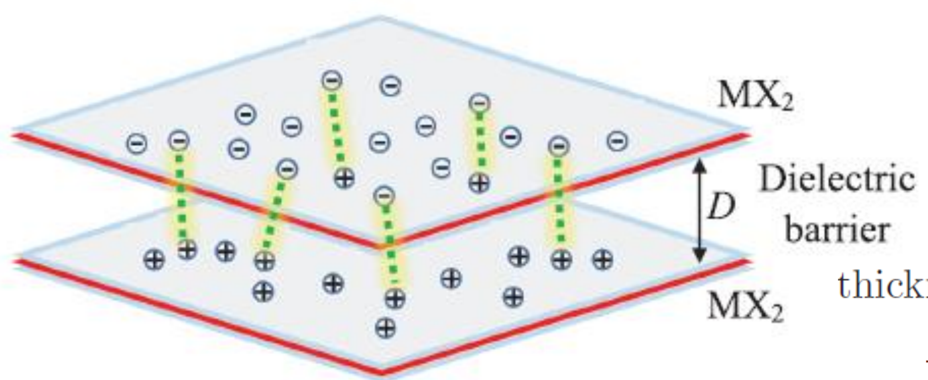
Spatially separated electrons and holes in a TMDC double layer

M is a transition metal

X is a chalcogenide

1. Gap between valence and conduction bands: 1.6 -- 1.8 eV

2. No fluctuations of the width of TMDC layers.



***h*-BN monolayers**

N_L number of monolayers

$$D = N_L D_{hBN}$$

thickness of one h-BN monolayer

$$D_{hBN} = 0.333 \text{ nm}$$

A and B 2D dipolar excitons in a TMDC double layer

The low-energy effective two-band single-electron Hamiltonian in the form of a spinor with a gapped spectrum for TMDCs in the *k* · *p* approximation:

D. Xiao, G.-B. Liu, W. Feng, X. Xu, and W. Yao, Phys. Rev. Lett. 108, 196802 (2012).

$$\hat{H}_s = at(\tau k_x \hat{\sigma}_x + k_y \hat{\sigma}_y) + \frac{\Delta}{2}\hat{\sigma}_z - \lambda\tau\frac{\hat{\sigma}_z - 1}{2}\hat{s}_z.$$

Strong spin-orbit coupling (SOC) !!!

Δ = 1.6 -- 1.8 eV

2λ = 0.1 -- 0.5 eV

$\hat{\sigma}$ denotes the Pauli matrices, a is the lattice constant, t is the effective hopping integral, Δ is the energy gap, $\tau = \pm 1$ is the valley index, 2λ is the spin splitting at the valence band top caused by the spin-orbit coupling (SOC), and $\hat{s}_z$ is the Pauli matrix for spin tl

T. C. Berkelbach, M. S. Hybertsen, and D. R. Reichman, Phys. Rev. B 88, 045318 (2013):

Significant spin-orbit splitting in the valence band leads to the formation of TMDC layers A and B in TMDC layers.

Type A excitons are formed by spin-up electrons from conduction and spin-down holes from valence bands.

Type B excitons are formed by spin-down electrons from conduction and spin-up holes from valence bands.

O.L. Berman and R. Ya. Kezerashvili, Phys. Rev. B 93, 245410 (2016): Analytical approach for a two-body problem

TABLE I. Effective masses of A and B excitons for different TMDC materials in units of the free electron mass at the interlayer separation $D = 5$ nm.

	Mass of exciton					
Exciton type	MoS_2	$MoSe_2$	$MoTe_2$	WS_2	WSe_2	WTe_2
A	0.499	0.555	0.790	0.319	0.345	0.277
B	0.545	0.625	0.976	0.403	0.457	0.501

Two-body problem in Keldysh and Coulomb potentials

$M_A = m_{e\uparrow} + m_{h\downarrow}$

$M_B = m_{e\downarrow} + m_{h\uparrow}$

Electron and hole effective masses are taken from

A. Kormanyos, V. Zolyomi, N. D. Drummond, and G. Burkard, Phys. Rev. X 4, 011034 (2014).

I. Kyläanpää and H.-P. Komsa, Phys. Rev. B 92, 205418 (2015).

Keldysh potential (screening effects):

$$V(r_{eh}) = -\frac{\pi k e^2}{(\varepsilon_1+\varepsilon_2)\rho_0}\left[H_0\left(\frac{r_{eh}}{\rho_0}\right) - Y_0\left(\frac{r_{eh}}{\rho_0}\right)\right].$$

$\varepsilon_1 = \varepsilon_2 = \varepsilon_d.$

$$r_{eh} = \sqrt{r^2 + D^2}$$

Coulomb potential (large $D \gg \rho_0$):

$$V(r) = -\frac{ke^2}{\varepsilon_d\sqrt{r^2+D^2}}$$

$\varepsilon_d = 4.89$ screening length $\rho_0 = 2\pi\zeta/\left[(\varepsilon_1+\varepsilon_2)/2\right]$, where ζ is the 2D polarizability.

h-BN monolayers $\zeta = 4.1$ Å

Polarizability is taken from T. C. Berkelbach, M. S. Hybertsen, and D. R. Reichman, Phys. Rev. B 88, 045318 (2013).

$H_0(x)$ and $Y_0(x)$ are Struve and Bessel functions of the second kind of order $\nu = 0$

Analytical approach (2D harmonic oscillator approximation): $r \ll D$ $\quad V(r) = -V_0 + \gamma r^2$

Keldysh potential:

$$V_0 = \frac{\pi k e^2}{2\varepsilon_d\rho_0}\left[H_0\left(\frac{D}{\rho_0}\right) - Y_0\left(\frac{D}{\rho_0}\right)\right],$$

$$\gamma = -\frac{\pi k e^2}{4\varepsilon_d\rho_0^2 D}\left[H_{-1}\left(\frac{D}{\rho_0}\right) - Y_{-1}\left(\frac{D}{\rho_0}\right)\right].$$

Coulomb potential:

$$V_0 = \frac{ke^2}{\varepsilon_d D}, \qquad \gamma = \frac{ke^2}{2\varepsilon_d D^3}$$

A single dipolar exciton

O. L. Berman and R. Ya. Kezerashvili, Physical Review B 96, 094502 (2017).

Energy spectrum: $$E_{NL} \equiv E_{e(h)} = -V_0 + (2N + 1 + |L|)\hbar \left(\frac{2\gamma}{\mu}\right)^{1/2}$$

$N = \min(\tilde{n}, \tilde{n}')$, $L = \tilde{n} - \tilde{n}'$, $\tilde{n}$, $\tilde{n}' = 0, 1, 2, 3, \ldots$ are the quantum numbers

the reduced mass $\mu = m_e m_h / (m_e + m_h)$

Ground state energy: $$E_{00} = -V_0 + \hbar \left(\frac{2\gamma}{\mu}\right)^{1/2}$$

The harmonic oscillator approximation holds for $D \gg D_0$

Exciton		MoS_2	$MoSe_2$	WS_2	WSe_2
A	Mass/m_0	1.1	1.33	0.84	0.93
	D_0, Å	0.29	0.23	0.32	0.30
B	Mass/m_0	0.98	1.15	0.62	0.66
	D_0, Å	0.31	0.25	0.37	0.36

D_0 for A excitons is smaller than for B excitons

The energy spectrum of the center-of-mass of the A (or B) dipolar exciton $\varepsilon_0^{A(B)}(\mathbf{P})$ is given by

$$\varepsilon_0^{A(B)}(\mathbf{P}) = \frac{P^2}{2M_{A(B)}}.$$

Binding energy of a dipolar exciton

O. L. Berman and R. Ya. Kezerashvili, Physical Review B 96, 094502 (2017).

N_L h-BN	Binding energy of A exiton, eV				Binding energy of B exiton, eV			
	MoS_2	$MoSe_2$	WS_2	WSe_2	MoS_2	$MoSe_2$	WS_2	WSe_2
8	0.049	0.054	0.040	0.041	0.045	0.050	0.029	0.030
10	0.045	0.049	0.038	0.040	0.043	0.046	0.028	0.029
12	0.042	0.044	0.036	0.038	0.039	0.042	0.028	0.029

A excitons:

N_L h-BN	Electron layer	MoS_2			$MoSe_2$			WS_2			WSe_2		
	Hole layer	$MoSe_2$	WS_2	WSe_2	MoS_2	WS_2	WSe_2	MoS_2	$MoSe_2$	WSe_2	MoS_2	$MoSe_2$	WS_2
8	B, eV	0.050	0.045	0.047	0.053	0.049	0.050	0.042	0.043	0.045	0.042	0.034	0.046
10	B, eV	0.046	0.043	0.044	0.048	0.045	0.046	0.041	0.041	0.043	0.039	0.033	0.044
12	B, eV	0.042	0.039	0.040	0.044	0.042	0.042	0.038	0.038	0.041	0.036	0.032	0.042

B excitons:

N_L h-BN	Electron layer	MoS_2			$MoSe_2$			WS_2			WSe_2		
	Hole layer	$MoSe_2$	WS_2	WSe_2	MoS_2	WS_2	WSe_2	MoS_2	$MoSe_2$	WSe_2	MoS_2	$MoSe_2$	WS_2
8	B, eV	0.046	0.038	0.038	0.049	0.041	0.041	0.035	0.035	0.031	0.034	0.027	0.034
10	B, eV	0.043	0.037	0.037	0.045	0.039	0.039	0.034	0.035	0.033	0.033	0.027	0.035
12	B, eV	0.040	0.035	0.035	0.041	0.037	0.037	0.033	0.034	0.032	0.032	0.026	0.034

Binding energy for A excitons is greater than for B excitons for the same monolayers.

The highest binding energy is in a $MoSe_2$ double layer, the lowest is in a WS_2 double layer.

Hamiltonian of two-component weakly interacting Bose gas of A and B dipolar excitons in a TMDC bilayer

O.L. Berman and R.Ya. Kezerashvili, Physical Review B 93, 245410 (2016); 96, 094502 (2017).

Weakly interacting Bose gas of the dipolar excitons at low densities: $na^2 \ll 1 \ (n \ll D^{-2})$

The Hamiltonian $\hat{H}$ of the 2D A and B interacting dipolar excitons is given by

$$\hat{H} = \hat{H}_A + \hat{H}_B + \hat{H}_I,$$

where $\hat{H}_{A(B)}$ are the Hamiltonians of A (B) excitons given by

$$\hat{H}_{A(B)} = \sum_{\mathbf{k}} E_{A(B)}(k) a^{\dagger}_{\mathbf{k}A(B)} a_{\mathbf{k}A(B)} + \frac{g_{AA(BB)}}{2S} \times \sum_{\mathbf{klm}} a^{\dagger}_{\mathbf{k}A(B)} a^{\dagger}_{\mathbf{l}A(B)} a_{A(B)\mathbf{m}} a_{A(B)\mathbf{k}+\mathbf{l}-\mathbf{m}},$$

and $\hat{H}_I$ is the Hamiltonian of the interaction between A and B excitons given by

$$\hat{H}_I = \frac{g_{AB}}{S} \sum_{\mathbf{klm}} a^{\dagger}_{\mathbf{k}A} a^{\dagger}_{\mathbf{l}B} a_{B\mathbf{m}} a_{A\mathbf{k}+\mathbf{l}-\mathbf{m}},$$

where $a^{\dagger}_{\mathbf{k}A(B)}$ and $a_{\mathbf{k}A(B)}$ are Bose creation and annihilation operators for A (B) dipolar excitons with the wave vector $\mathbf{k}$, S is the area of the system,

We assume:

exciton-exciton dipole-dipole repulsion exists only at distances between excitons greater than distance from the exciton to the classical turning point.

The distance between two excitons cannot be less than this distance.

$$g_{AA} = g_{BB} = g_{AB} \equiv g$$

$g_{AA(BB)}$ and g_{AB} are the interaction constants

The collective excitations and sound velocity in a two-component dilute gas of A and B excitons

The Bogoliubov approximation for a two-component weakly interacting Bose gas: diagonalize the many-particle Hamiltonian.

The product of four operators is replaced by the product of two operators.

Most of the particles belong to BEC.

Only the interactions between the condensate and overcondensate particles are taken into account,

the interactions between overcondensate particles are neglected.

The condensate operators are replaced by numbers.

The resulting Hamiltonian is quadratic with respect to the creation and annihilation operators.

Spectrum of collective excitations: O.L. Berman and R.Ya. Kezerashvili, Physical Review B 93, 245410 (2016)

$$\varepsilon_j(k) = \sqrt{\frac{\omega_A^2(k) + \omega_B^2(k) + (-1)^{j-1}\sqrt{\left[\omega_A^2(k) - \omega_B^2(k)\right]^2 + 4g^2n^2\varepsilon_{(0)A}(k)\varepsilon_{(0)B}(k)}}{2}},$$

$$\omega_A(k) = \sqrt{\varepsilon_{(0)A}^2(k) + gn\varepsilon_{(0)A}(k)}, \quad \omega_B(k) = \sqrt{\varepsilon_{(0)B}^2(k) + gn\varepsilon_{(0)B}(k)}.$$

Reduced mass of two excitons

j= 1,2

$$n_A = n_B = n/2$$

$$\mu_{AB} = M_A M_B/(M_A + M_B).$$

$$c_j = \sqrt{\frac{gn}{2}\left(\frac{1}{2M_A} + \frac{1}{2M_B} + (-1)^{j-1}\sqrt{\left(\frac{1}{2M_A} - \frac{1}{2M_B}\right)^2 + \frac{1}{M_A M_B}}\right)}.$$

$$c = \sqrt{\frac{gn}{2}\left(\frac{1}{M_A} + \frac{1}{M_B}\right)}$$

c is a sound velocity

c for another branch is zero

Superfluidity of two-component A and B dipolar excitons in a TMDC bilayer

O.L. Berman and R.Ya. Kezerashvili, Physical Review B 93, 245410 (2016); 96, 094502 (2017).

The mean total current of 2D excitons in the coordinate system, moving with a velocity *u*

$$\mathbf{J} = \int \frac{d^2p}{(2\pi\hbar)^2}\mathbf{p}\{f[\varepsilon_1(p) - \mathbf{pu}] + f[\varepsilon_2(p) - \mathbf{pu}]\},$$

where $f[\varepsilon_1(p)] = \{\exp[\varepsilon_1(p)/(k_BT)] - 1\}^{-1}$ and $f[\varepsilon_2(p)] = \{\exp[\varepsilon_2(p)/(k_BT)] - 1\}^{-1}$ are the Bose-Einstein distribution

$$\mathbf{J} = \rho_n \mathbf{u}.$$

ρ_n is the density of the normal component

ρ_s is the density of the superfluid component

$$\rho_n(T) = \frac{3\zeta(3)}{2\pi\hbar^2}k_B^3T^3\left(\frac{1}{c_1^4} + \frac{1}{c_2^4}\right) \quad k_BT \ll M_{A(B)}c_j^2$$

$$\rho_n(T_c) = \rho = M_An_A + M_Bn_B$$

$$T_c = \left[\frac{2\pi\hbar^2\rho}{3\zeta(3)k_B^3\left(\frac{1}{c_1^4} + \frac{1}{c_2^4}\right)}\right]^{1/3}$$

$$n_A = n_B = n/2 \quad T_c = \left[\frac{2\pi\hbar^2\rho c^4}{3\zeta(3)k_B^3}\right]^{1/3}$$

For one-component system in CQWs or gapped graphene:

$Q_A = 8/M_A$ or $Q_B = 8/M_B$

n/s $s=4$ is the spin degeneracy factor

For two-component system in TMDC:

***Tc* is a mean field critical temperature**

$$T_c = \frac{1}{k_B}\left[\frac{\pi\hbar^2 g^2 n^3}{12\zeta(3)}Q\right]^{1/3}$$

No spin degeneracy

$$Q = \frac{M_A + M_B}{(\mu_{AB})^2}$$

For a one-component dilute exciton gas $Q^{1/3}n/s$ is always less than $Q^{1/3}n$ for a two-component Bose gas of A and B dipolar excitons. Therefore, *Tc* is always higher for a two-component dilute dipolar exciton gas in TMDC than for a one-component dilute dipolar exciton gas in CQWs and gapped graphene.

Q factor and critical temperature T_c

O. L. Berman and R. Ya. Kezerashvili, Physical Review B 96, 094502 (2017).

Keldysh potential:

	MoS_2	$MoSe_2$	WS_2	WSe_2
Q, $[1/m_0]$	7.74	6.52	11.47	10.67
μ_{AB}, $[m_0]$	0.52	0.62	0.36	0.39
$M_A + M_B$, $[m_0]$	2.08	2.48	1.46	1.49

Electron layer	MoS_2			$MoSe_2$			WS_2			WSe_2		
Hole layer	$MoSe_2$	WS_2	WSe_2	MoS_2	WS_2	WSe_2	MoS_2	$MoSe_2$	WSe_2	MoS_2	$MoSe_2$	WS_2
Q, $[1/m_0]$	7.31	9.25	9.05	6.86	8.03	7.88	9.17	8.57	11.2	8.80	8.25	10.9
μ_{AB}, $[m_0]$	0.55	0.44	0.45	0.59	0.50	0.52	0.44	0.47	0.37	0.46	0.49	0.38
$M_A + M_B$, $[m_0]$	2.21	1.77	1.82	2.35	2.04	2.09	1.77	1.90	1.51	1.85	1.98	1.54

Critical Temperature Tc (in K) :

$N_L = 10$ of h-BN monolayers.

Electron layer	MoS_2				$MoSe_2$				WS_2				WSe_2			
Hole layer	MoS_2	$MoSe_2$	WS_2	WSe_2	MoS_2	$MoSe_2$	WS_2	WSe_2	MoS_2	$MoSe_2$	WS_2	WSe_2	MoS_2	$MoSe_2$	WS_2	WSe_2
$n = 3 \times 10^{11}$ cm^{-2}	41	39	44	43	40	38	41	40	43	42	47	47	43	41	46	45
$n = 5 \times 10^{11}$ cm^{-2}	79	76	84	82	77	71	79	77	83	80	91	89	81	78	88	86
Q, $[1/m_0]$	7.74	7.31	9.25	9.05	7.54	6.52	8.03	7.88	9.17	8.57	11.47	11.2	8.80	8.25	10.9	10.67

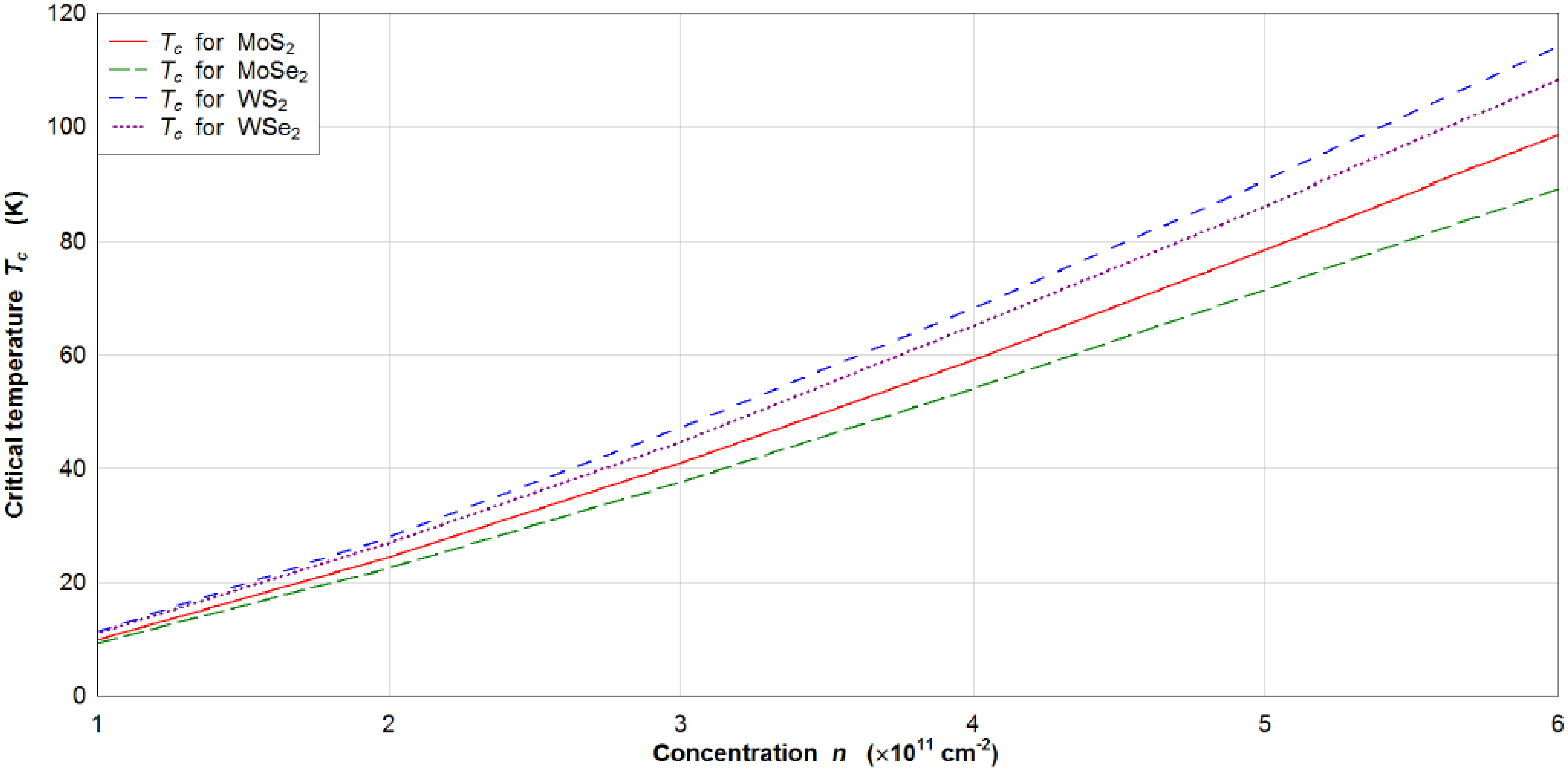
The largest Q and Tc are in a WS_2 double layer, the smallest Q and Tc are in a $MoSe_2$ double layer.

Critical Temperature

O. L. Berman and R. Ya. Kezerashvili, Physical Review B 96, 094502 (2017).

Keldysh potential:

$N_L = 10$ of h-BN monolayers.

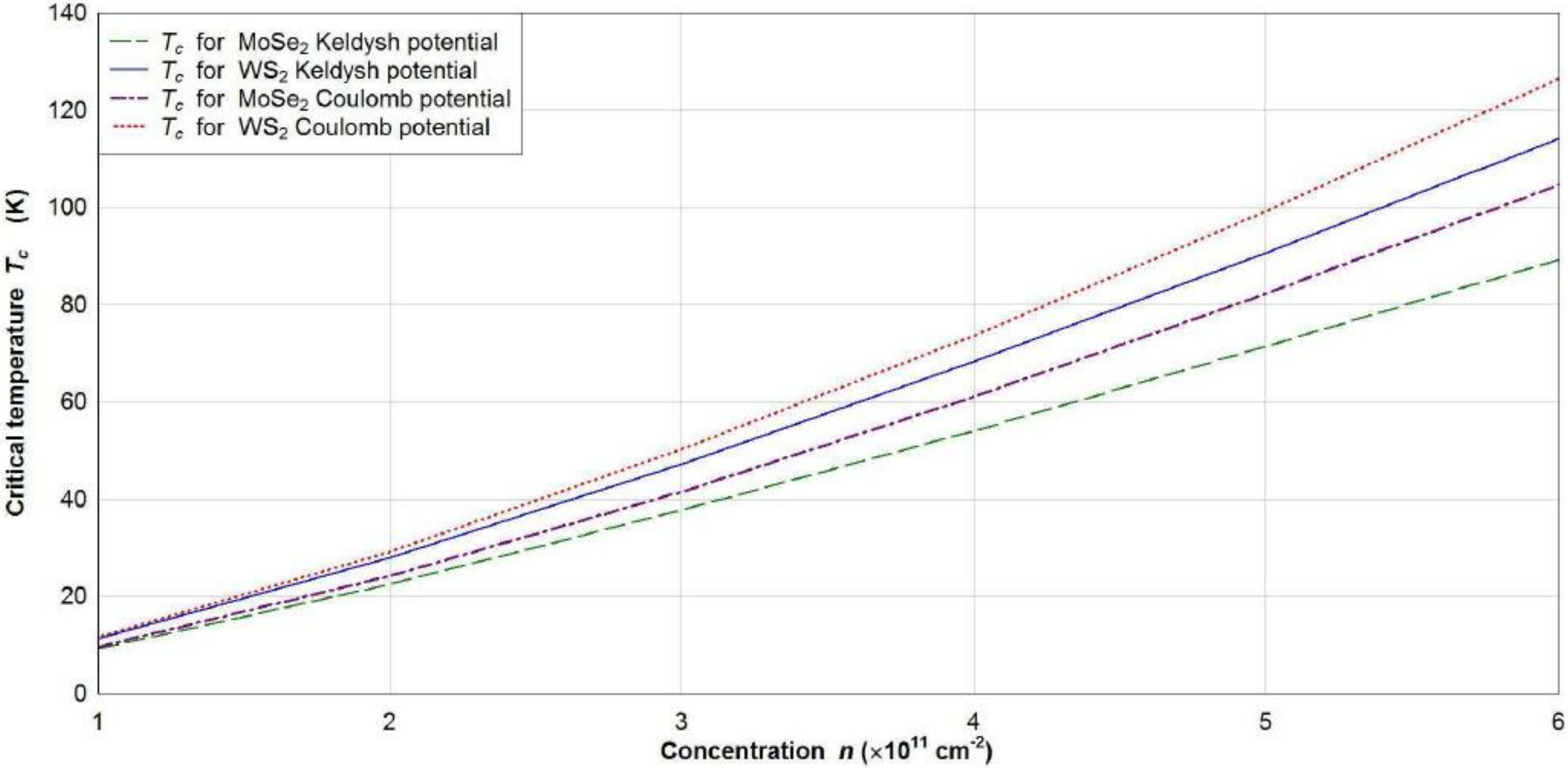


The largest *Tc i*is in a WS_2 double layer, the smallest *Tc* is in a $MoSe_2$ double layer.

Critical Temperature for Keldysh and Coulomb potentials

O. L. Berman and R. Ya. Kezerashvili, Physical Review B 96, 094502 (2017).

$N_L = 10$ of h-BN monolayers.

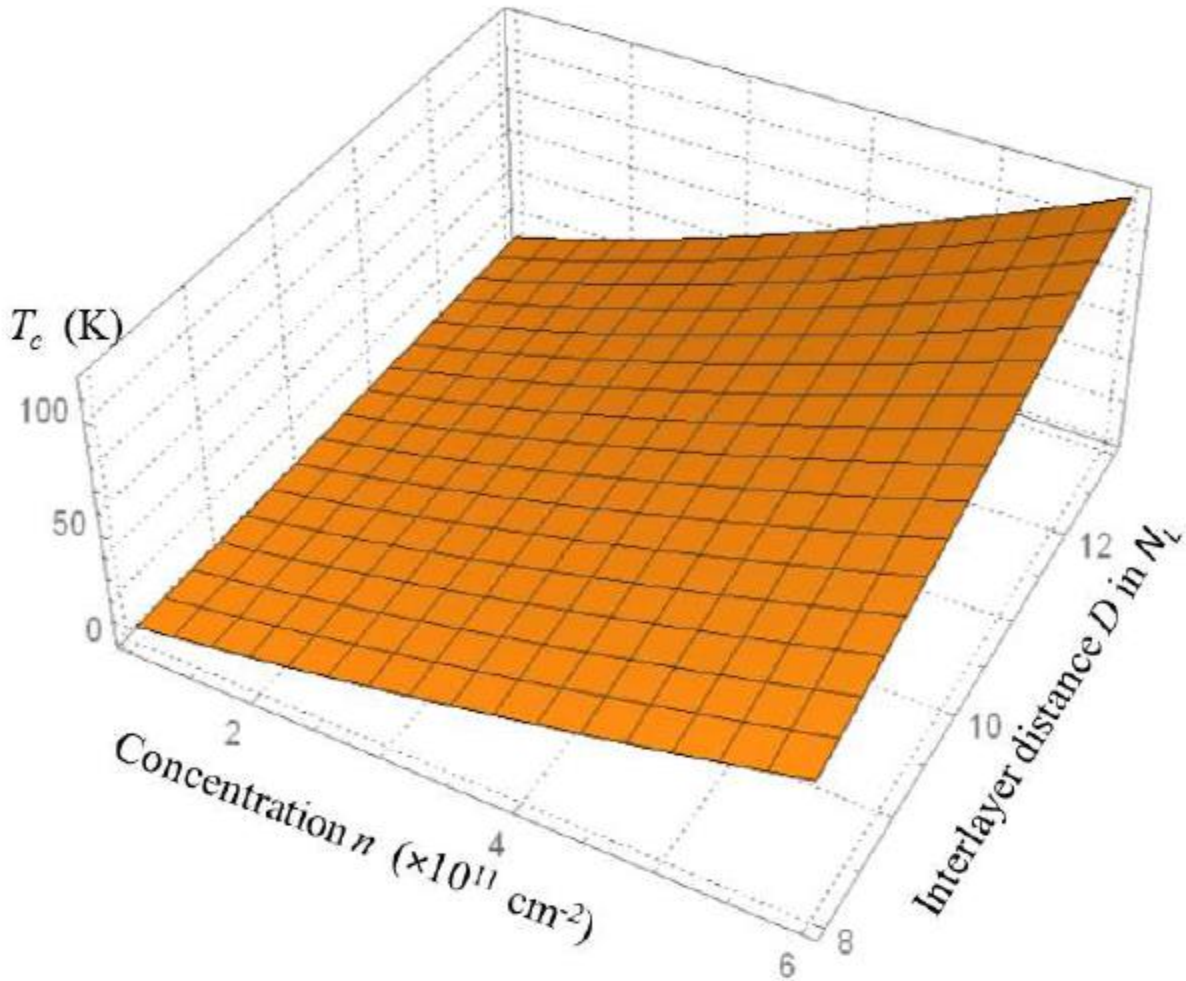


T_c for the Keldysh potential is smaller than for the Coulomb potential due to the screening effects

Critical Temperature for a WS_2 double layer

O. L. Berman and R. Ya. Kezerashvili, Physical Review B 96, 094502 (2017).

Keldysh potential:



Conclusions

O. L. Berman and R. Ya. Kezerashvili, Physical Review B 93, 245410 (2016); 96, 094502 (2017).

- We propose a physical realization to observe high-temperature superconducting electron-hole currents in two parallel TMDC layers, caused by the superfluidity of quasi-two-dimensional dipolar A and B excitons in a TMDC double layer.

- The binding energies for A and B dipolar excitons in various TMDC double layers, taking into account screening effects by employing the approximated Keldysh potential, were obtained.

- The spectrum of collective excitations, obtained for a TMDC bilayer, is characterized by two branches due to the fact that the exciton system under consideration is a two-component weakly interacting Bose gas of A and B excitons in a TMDC double layer.

- The mean field critical temperature for the phase transition is analyzed for various TMDC materials.

- The mean field phase transition temperature for two-component dipolar excitons in a TMDC double layer is about one order of magnitude higher than for any one-component exciton system of semiconductor CQWs or gapped graphene.

2D electrons and holes in a phosphorene monolayer

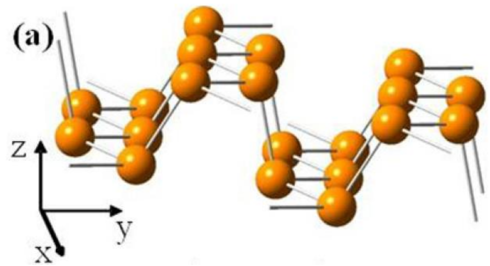
Phosphorene is an atom-thick layer of black phosphorus

- natural band gap   *Δ ≈ 2.2 eV*
- high hole mobility
- prominent anisotropic electron and hole effective masses, carrier mobility
- large excitonic binding energy ~ *0.9 eV*

A. H. Woomer et al, ACS Nano 9, 8869 (2015).

X. Wang et al, Nature Nanotechnology 10, 517 (2015).

X. Peng, Q. Wei, and A. Copple, Phys. Rev. B **90**, 085402 (2014).



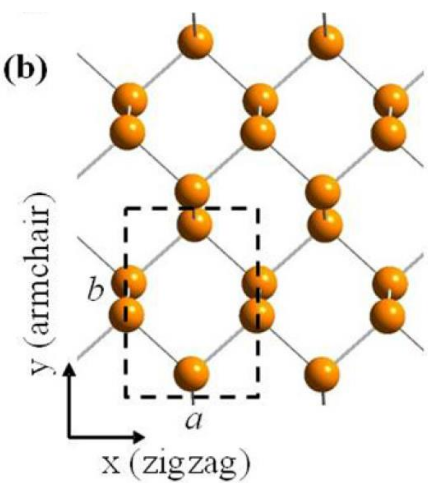
The single electron and hole energy spectrum for a phosphorene monolayer:

$$\varepsilon_l^{(0)}(\mathbf{p}) = \frac{p_x^2}{2m_x^l} + \frac{p_y^2}{2m_y^l}, \quad l = e, h,$$

A. S. Rodin, A. Carvalho, and A. H. Castro Neto, Phys. Rev. B **90**, 075429 (2014).

L. Seixas, A. S. Rodin, A. Carvalho, and A. H. Castro Neto, Phys. Rev. B **91**, 115437 (2015).

where m_x^l and m_y^l are the electron/hole effective masses along the x and y directions, respectively. We assume that the OX and OY axes correspond to the armchair and zigzag directions in a phosphorene monolayer, respectively.



Electron and hole effective masses are taken from

[35] X. Peng, Q. Wei, and A. Copple, Phys. Rev. B **90**, 085402 (2014).

[36] V. Tran and L. Yang, Phys. Rev. B **89**, 245407 (2014).

[37] C. J. Páez, K. DeLello, D. Le, A. L. C. Pereira, and E. R. Mucciolo, Phys. Rev. B **94**, 165419 (2016).

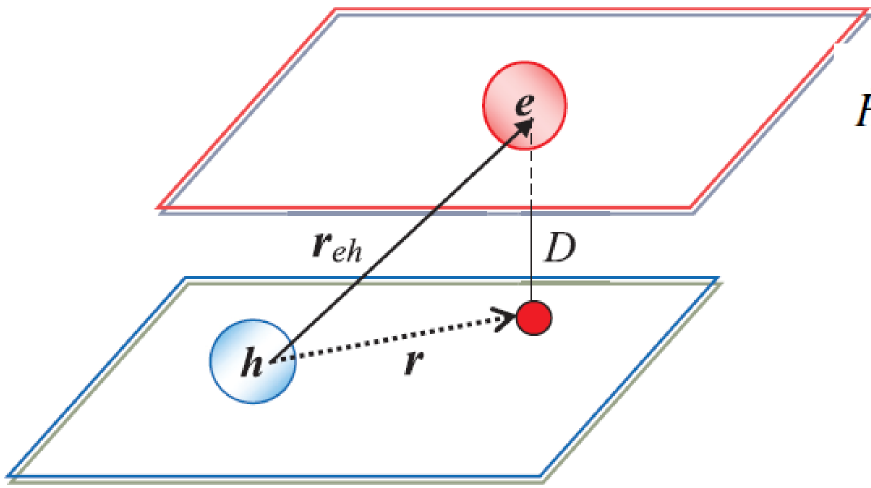
[38] J. Qiao, X. Kong, Z.- X. Hu, F. Yang, and W. Ji, Nat. Commun. **5**, 4475 (2014).

X. Peng, Q. Wei, and A. Copple, Phys. Rev. B **90**, 085402 (2014).

A single dipolar exciton in a phosphorene double layer

O.L. Berman, G. Gumbs, and R.Ya. Kezerashvili, Physical Review B 96, 014505 (2017)

A dipolar exciton consisting of a spatially separated electron and hole in a phosphorene double layer.



Hamiltonian

$$\hat{H}_0 = -\frac{\hbar^2}{2m_x^e}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m_y^e}\frac{\partial^2}{\partial y_1^2} - \frac{\hbar^2}{2m_x^h}\frac{\partial^2}{\partial x_2^2} - \frac{\hbar^2}{2m_y^h}\frac{\partial^2}{\partial y_2^2} + V(\sqrt{r^2 + D^2}),$$

wave function $\Psi(\mathbf{R},\mathbf{r}) = e^{i\mathbf{P}\cdot\mathbf{R}/\hbar}\varphi(\mathbf{r})$

the center of mass of an electron-hole pair $\mathbf{R} = (X, Y)$

$$X = (m_x^e x_1 + m_x^h x_2)/(m_x^e + m_x^h),\ Y = (m_x^e y_1 + m_x^h y_2)/(m_x^e + m_x^h),$$

the relative motion of an electron-hole pair

$$\mathbf{r} = (x, y), \quad x = x_1 - x_2,\ y = y_1 - y_2,\ r^2 = x^2 + y^2$$

Schrödinger equation:

$$\left[-\frac{\hbar^2}{2\mu_x}\frac{\partial^2}{\partial x^2} - \frac{\hbar^2}{2\mu_y}\frac{\partial^2}{\partial y^2} + V(\sqrt{r^2+D^2})\right]\varphi(x,y) = \mathcal{E}\varphi(x,y)$$

the reduced masses

$$\mu_x = \frac{m_x^e m_x^h}{m_x^e + m_x^h} \text{ and } \mu_y = \frac{m_y^e m_y^h}{m_y^e + m_y^h}$$

Two-body problem in Keldysh and Coulomb potentials

Insulator between two phosphorene monolayers: ***h*-BN monolayers**

N_L number of monolayers

$$D = N_L D_{hBN}$$

thickness of one h-BN monolayer

$$D_{hBN} = 0.333 \text{ nm}$$

Keldysh potential (screening effects):

$$V(r_{eh}) = -\frac{\pi k e^2}{(\varepsilon_1 + \varepsilon_2)\rho_0}\left[H_0\left(\frac{r_{eh}}{\rho_0}\right) - Y_0\left(\frac{r_{eh}}{\rho_0}\right)\right].$$

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_d.$$

$$r_{eh} = \sqrt{r^2 + D^2}$$

Coulomb potential (large $D \gg \rho_0$):

$$V(r) = -\frac{ke^2}{\varepsilon_d\sqrt{r^2 + D^2}}$$

$\varepsilon_d = 4.89$ screening length $\rho_0 = 2\pi\zeta/\left[(\varepsilon_1 + \varepsilon_2)/2\right]$, where ζ is the 2D polarizability.

***h*-BN monolayers** $\zeta = 4.1$ Å **Polarizability is taken from** A. S. Rodin, A. Carvalho, and A. H. Castro Neto, Phys. Rev. B **90**, 075429 (2014).

$H_0(x)$ and $Y_0(x)$ are Struve and Bessel functions of the second kind of order $\nu = 0$

Analytical approach (2D harmonic oscillator approximation): $r \ll D$ $\quad V(r) = -V_0 + \gamma r^2$

Keldysh potential:

$$V_0 = \frac{\pi k e^2}{2\varepsilon_d \rho_0}\left[H_0\left(\frac{D}{\rho_0}\right) - Y_0\left(\frac{D}{\rho_0}\right)\right],$$

$$\gamma = -\frac{\pi k e^2}{4\varepsilon_d \rho_0^2 D}\left[H_{-1}\left(\frac{D}{\rho_0}\right) - Y_{-1}\left(\frac{D}{\rho_0}\right)\right].$$

Coulomb potential:

$$V_0 = \frac{ke^2}{\varepsilon_d D}, \qquad \gamma = \frac{ke^2}{2\varepsilon_d D^3}$$

Effective masses and binding energy of a dipolar exciton

O.L. Berman, G. Gumbs, and R.Ya. Kezerashvili, Physical Review B 96, 014505 (2017)

Energy spectrum of the center of mass of a single dipolar exciton:

$$\varepsilon_0(\mathbf{P}) = \frac{P_x^2}{2M_x} + \frac{P_y^2}{2M_y} \qquad \varepsilon_0(\mathbf{P}) = \varepsilon_0(P,\Theta) = \frac{P^2}{2M_0(\Theta)}$$

$P_x = P\cos\Theta$ $P_y = P\sin\Theta$ $M_x = m_x^e + m_x^h$ and $M_y = m_y^e + m_y^h$ are the effective exciton masses, $M_0(\Theta)$ is the effective angle-dependent exciton mass

$$M_0(\Theta) = \left[\frac{\cos^2\Theta}{M_x} + \frac{\sin^2\Theta}{M_y}\right]^{-1}$$

TABLE II. The critical temperatures under the assumption about the soundlike spectrum of collective excitations for different sets of masses from Refs. [35–38]. The phosphorene layers are separated by 7 layers of h-BN. μ_0 and $M_x M_y$ are expressed in units of free electron mass m_0 and m_0^2, respectively.

	[35]	[36]	[37]	[38]
μ_0 ($\times 10^{-2} m_0$)	3.99	4.11	4.84	4.79
Coulomb potential T_c (K)	182	192	174	172
Keldysh potential T_c (K)	115	121	109	107
$M_x M_y$ ($\times m_0^2$)	1.67	1.23	2.24	2.39

Binding energy:

For the number $N_L = 7$ of h-BN monolayers, placed between two phosphorene monolayers, the binding energies of dipolar excitons, calculated for the sets of the masses from Refs. [35–38] by using Eq. (13), are given by 28.2 meV, 29.6 meV, 37.6 meV, and 37.2 meV.

Hamiltonian of a weakly interacting Bose gas of dipolar excitons in a phosphorene double layer

O.L. Berman, G. Gumbs, and R.Ya. Kezerashvili, Physical Review B 96, 014505 (2017)

Weakly interacting Bose gas of the dipolar excitons at low concentrations:

$na^2 \ll 1 \; (n \ll D^{-2})$

The model Hamiltonian $\hat{H}$ of the 2D interacting dipolar excitons is given by

$$\hat{H} = \sum_{\mathbf{P}} \varepsilon_0(P,\Theta) a_{\mathbf{P}}^{\dagger} a_{\mathbf{P}} + \frac{g}{S} \sum_{\mathbf{P}_1\mathbf{P}_2\mathbf{P}_3} a_{\mathbf{P}_1}^{\dagger} a_{\mathbf{P}_2}^{\dagger} a_{\mathbf{P}_3} a_{\mathbf{P}_1+\mathbf{P}_2-\mathbf{P}_3},$$

where $a_{\mathbf{P}}^{\dagger}$ and $a_{\mathbf{P}}$ are Bose creation and annihilation operators for dipolar excitons with momentum $\mathbf{P}$, S is a normalization area for the system, $\varepsilon_0(P,\Theta)$ is the angular-dependent energy spectrum of noninteracting dipolar excitons,

We assume:

exciton-exciton dipole-dipole repulsion exists only at distances between excitons greater than distance from the exciton to the classical turning point.

The distance between two excitons cannot be less than this distance.

, we obtain the chemical potential μ of the entire exciton system by minimizing $\hat{H}_0 - \mu\hat{N}$ with respect to the 2D concentration n, where $\hat{N}$ denotes the number operator.

The Bogoliubov approximation for a weakly interacting Bose gas: diagonalize the many-particle Hamiltonian.

The product of four operators is replaced by the product of two operators.

Most of the particles belong to BEC.

the chemical potential μ $\quad \mu = gn$

Only the interactions between the condensate and noncondensate particles are taken into account,

the interactions between noncondensate particles are neglected.

The condensate operators are replaced by numbers.

The resulting Hamiltonian is quadratic with respect to the creation and annihilation operators.

The collective excitations and sound velocity in a dilute gas of dipolar excitons in phosphorene double layer

O.L. Berman, G. Gumbs, and R.Ya. Kezerashvili, Physical Review B 96, 014505 (2017)

Hamiltonian $\hat{H}_{\rm col}$ of the collective excitations in the Bogoliubov approximation for the weakly interacting gas of dipolar excitons in phosphorene is given by

$$\hat{H}_{\rm col} = \sum_{P\neq 0,\Theta} \varepsilon(P,\Theta)\alpha^{\dagger}_{\mathbf{P}}\alpha_{\mathbf{P}},$$

where $\alpha^{\dagger}_{j\mathbf{P}}$ and $\alpha_{j\mathbf{P}}$ are the creation and annihilation Bose operators for the quasiparticles with the energy dispersion corresponding to the angular-dependent spectrum of the collective excitations $\varepsilon(P,\Theta)$, described by

$$\varepsilon(P,\Theta) = \{[\varepsilon_0(P,\Theta) + \mu]^2 - \mu^2\}^{1/2}.$$

In the limit of small momenta P, when $\varepsilon_0(P,\Theta) \ll gn$, $c_S(\Theta)$ is the angular-dependent sound velocity

$$\varepsilon(P,\Theta) = c_S(\Theta)P$$

$$c_S(\Theta) = \sqrt{\frac{gn}{M_0(\Theta)}}$$

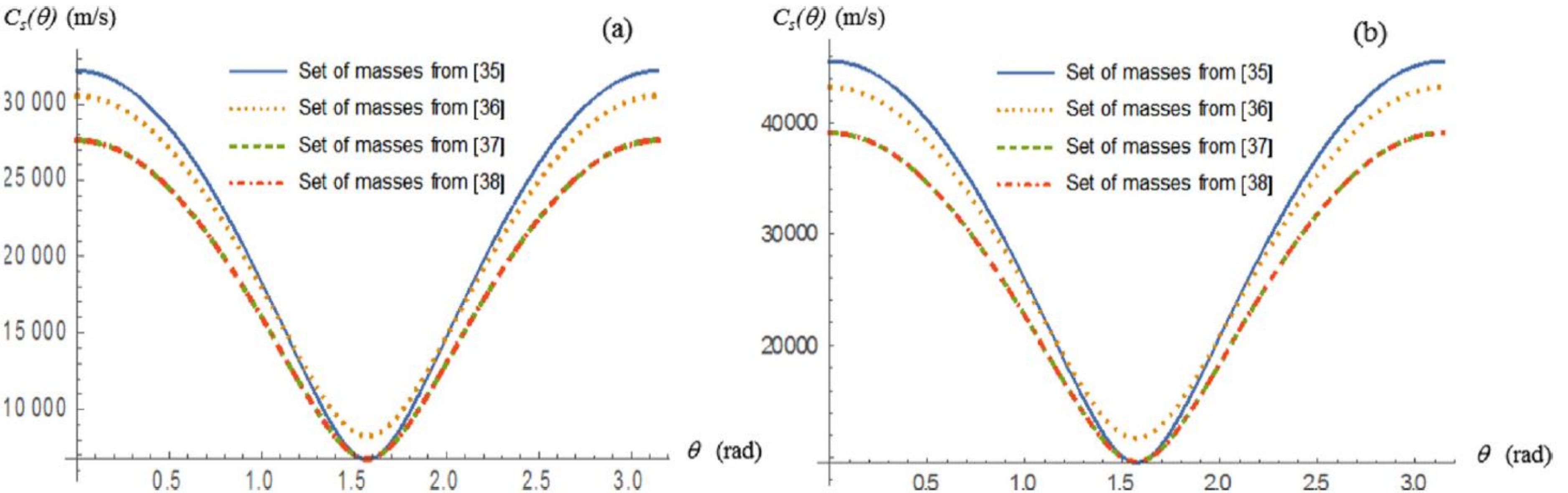
Since at small momenta the energy spectrum of the quasiparticles for a weakly interacting gas of dipolar excitons is soundlike, this means that the system satisfies the Landau criterion for superfluidity

The critical exciton velocity for superfluidity is angular dependent. $v_c(\Theta) = c_S(\Theta)$

Angular Dependent Sound Velocity

O.L. Berman, G. Gumbs, and R.Ya. Kezerashvili, Physical Review B 96, 014505 (2017)

Keldysh potential. Coulomb potential.



$$n = 2 \times 10^{16}\ \mathrm{m}^{-2}$$

number $N_L = 7$ of h-BN monolayers, placed between two phosphorene monolayers.

Directional Superfluidity of Dipolar Excitons in a Phosphorene Double Layer

O.L. Berman, G. Gumbs, and R.Ya. Kezerashvili, Physical Review B 96, 014505 (2017)

The mean total mass current of 2D excitons in the coordinate system, moving with a velocity *u*

$$\mathbf{J} = s \int \frac{d^2 P}{(2\pi\hbar)^2} \mathbf{P} f[\varepsilon(P,\Theta) - \mathbf{P}\mathbf{u}].$$

Spin degeneracy factor $s = 4$

$f[\varepsilon(P,\Theta)] = \{\exp[\varepsilon(P,\Theta)/(k_B T)] - 1\}^{-1}$ is the Bose-Einstein distribution function for quasiparticles

The normal density ρ_n in the anisotropic system has tensor form $J_i = \rho_n^{(ij)}(T) u_j$.

For an anisotropic BCS superconductor: W. M. Saslow, Phys. Rev. Lett. **31**, 870 (1973).

the tensor of the concentration of the normal component $n_n^{(ij)} = \rho_n^{(ij)}/M_i$

$$n_n^{(xx)}(T) = \frac{s}{k_B M_x T} \int_0^\infty dP \frac{P^3}{(2\pi\hbar)^2} \times \int_0^{2\pi} d\Theta \frac{\exp[\varepsilon(P,\Theta)/(k_B T)]}{\{\exp[\varepsilon(P,\Theta)/(k_B T)] - 1\}^2} \cos^2\Theta$$

$$n_n^{(xy)}(T) = 0, \qquad n_n^{(yx)}(T) = 0$$

$$\varepsilon(P,\Theta) = c_S(\Theta) P$$

$$n_n^{(yy)}(T) = \frac{s}{k_B M_y T} \int_0^\infty dP \frac{P^3}{(2\pi\hbar)^2} \times \int_0^{2\pi} d\Theta \frac{\exp[\varepsilon(P,\Theta)/(k_B T)]}{\{\exp[\varepsilon(P,\Theta)/(k_B T)] - 1\}^2} \sin^2\Theta,$$

$$n_n^{(xx)}(T) = n_n^{(yy)}(T) = \frac{2\zeta(3) s (k_B T)^3 \sqrt{M_x M_y}}{\pi(\hbar g n)^2}.$$

$$\varepsilon(P,\Theta) = \{[\varepsilon_0(P,\Theta) + \mu]^2 - \mu^2\}^{1/2}$$

For an anisotropic BCS superconductor: $n_n^{(xx)}(T) = n_n^{(yy)}(T)$

$n_n^{(xx)}(T) \neq n_n^{(yy)}(T)$ **Phosphorene double layer**

Angular Dependence of Mean-Field Critical Temperature for Superfluidity I

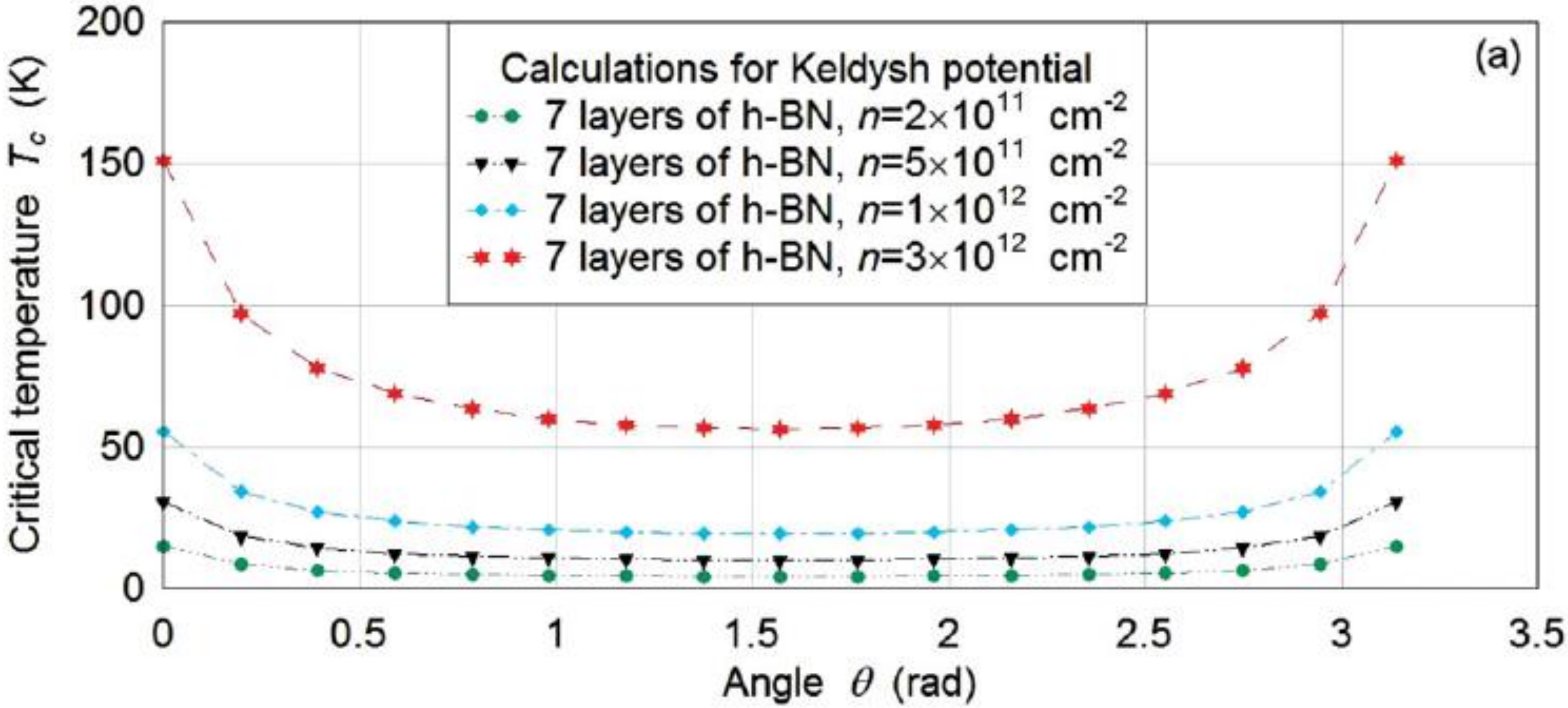
O.L. Berman, G. Gumbs, and R.Ya. Kezerashvili, Physical Review B 96, 014505 (2017)

$$\tilde{n}_n(\Theta,T) = \sqrt{\left[n_n^{(xx)}(T)\right]^2 \cos^2\Theta + \left[n_n^{(yy)}(T)\right]^2 \sin^2\Theta}$$

$$\tilde{n}_n(\Theta,T_c(\Theta)) = n$$

Keldysh potential.

$$\varepsilon(P,\Theta) = \{[\varepsilon_0(P,\Theta)+\mu]^2 - \mu^2\}^{1/2}$$



number $N_L = 7$ of h-BN monolayers, placed between two phosphorene monolayers.

The set of masses is taken from C. J. Páez, K. DeLello, D. Le, A. L. C. Pereira, and E. R. Mucciolo, Phys. Rev. B **94**, 165419 (2016).

Angular Dependence of Mean-Field Critical Temperature for Superfluidity II

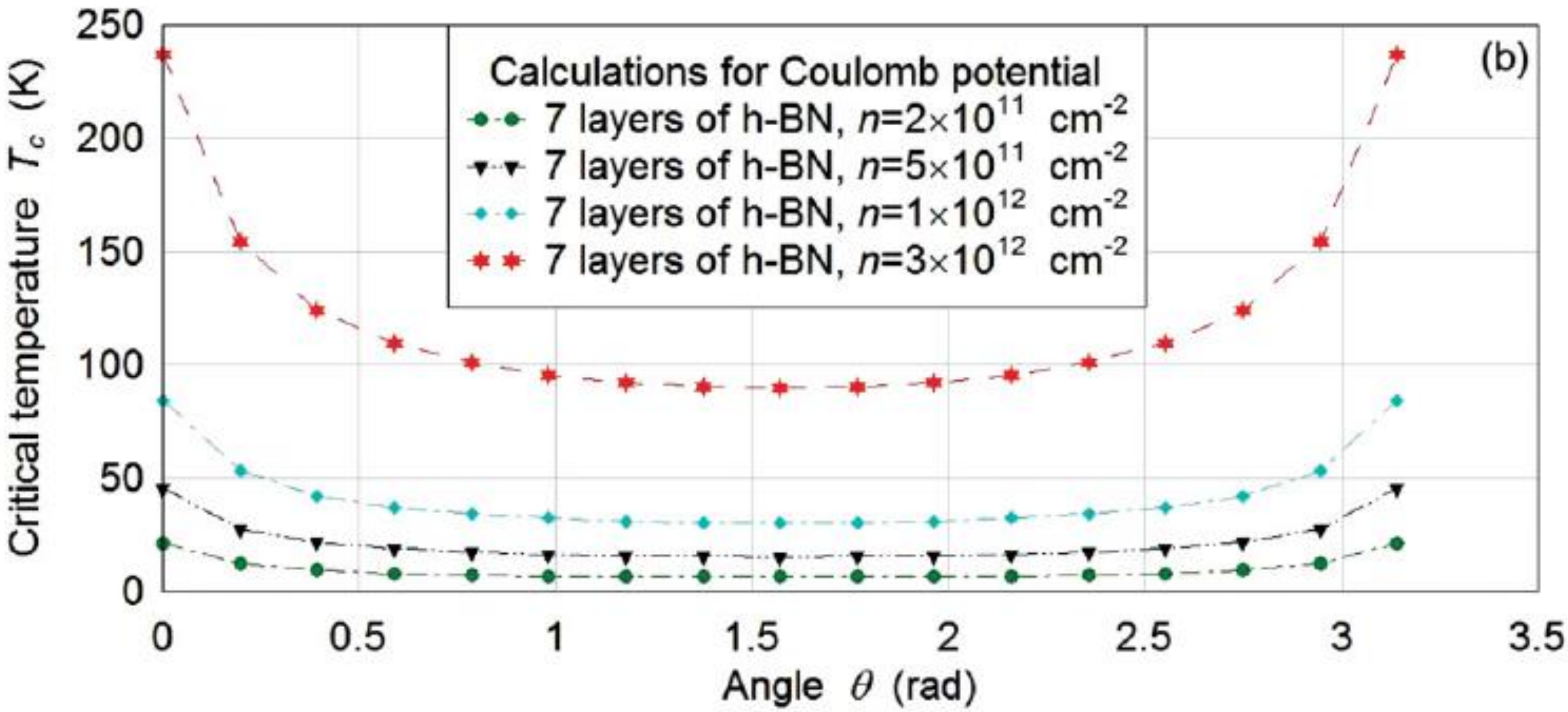
O.L. Berman, G. Gumbs, and R.Ya. Kezerashvili, Physical Review B 96, 014505 (2017)

$$\tilde{n}_n(\Theta,T) = \sqrt{\left[n_n^{(xx)}(T)\right]^2 \cos^2\Theta + \left[n_n^{(yy)}(T)\right]^2 \sin^2\Theta}$$

Coulomb potential.

$$\tilde{n}_n(\Theta, T_c(\Theta)) = n$$

$$\varepsilon(P,\Theta) = \{[\varepsilon_0(P,\Theta)+\mu]^2 - \mu^2\}^{1/2}$$



number $N_L = 7$ of h-BN monolayers, placed between two phosphorene monolayers.

The set of masses is taken from C. J. Páez, K. DeLello, D. Le, A. L. C. Pereira, and E. R. Mucciolo, Phys. Rev. B **94**, 165419 (2016).

Conclusions

O.L. Berman, G. Gumbs, and R.Ya. Kezerashvili, Physical Review B 96, 014505 (2017)

- The influence of the anisotropy of the dispersion relation of dipolar excitons in a double layer of phosphorene on the excitonic BEC and directional superfluidity has been investigated.

- The analytical expressions for the single dipolar exciton energy spectrum and wave functions have been derived.

- The anisotropy of the energy band structure in a phosphorene causes the critical velocity of the superfluidity to depend on the direction of motion of dipolar excitons.

- The dependence of the concentrations of the normal and superfluid components and the mean-field critical temperatures for superfluidity on the direction of motion of dipolar excitons occurs beyond the soundlike approximation for the spectrum of collective excitations.

- The directional superfluidity of dipolar excitons in a phosphorene double layer is possible.

Trapping Cavity Polaritons

D. W. Snoke, Science 298, 1368 (2002)

P. Littlewood, Science 316, 989 (2007)

D. W. Snoke and J. Keeling, Phys. Today 70, 54 (2017).

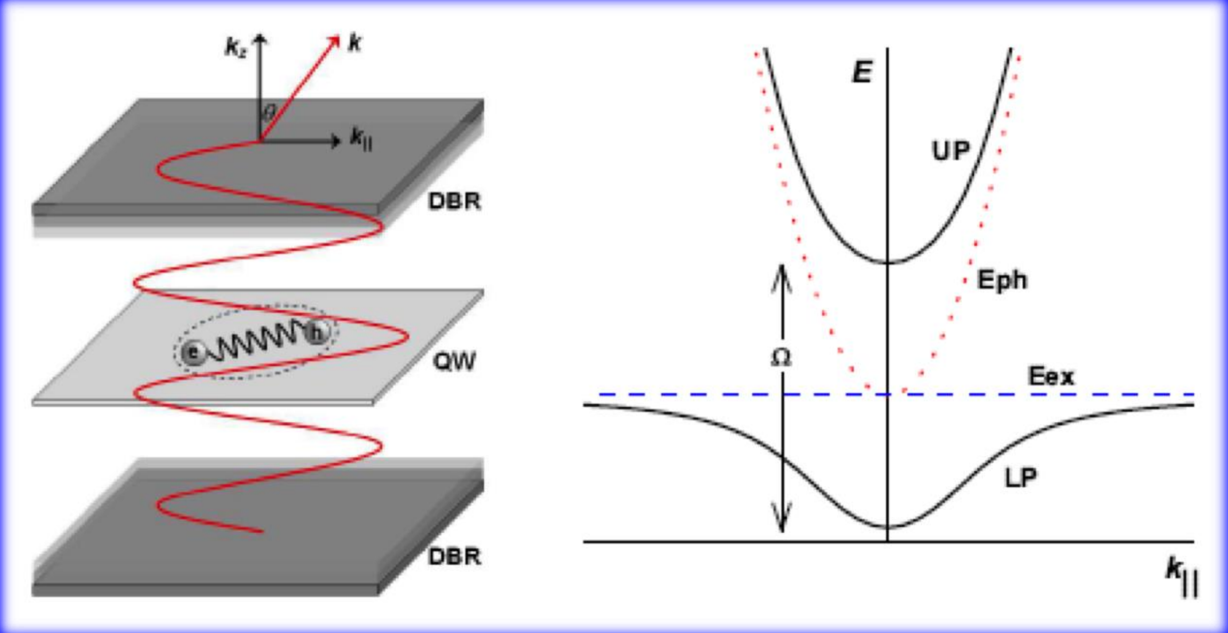
Cavity polaritons in TMDC:

Experiments:

X. Liu, et al., Nat. Photonics 9, 30 (2015).

L. C. Flatten, et al., Sci. Rep. 6, 33134 (2016).

S. Dufferwiel, et al., Nature Commun. 6, 8579 2015).



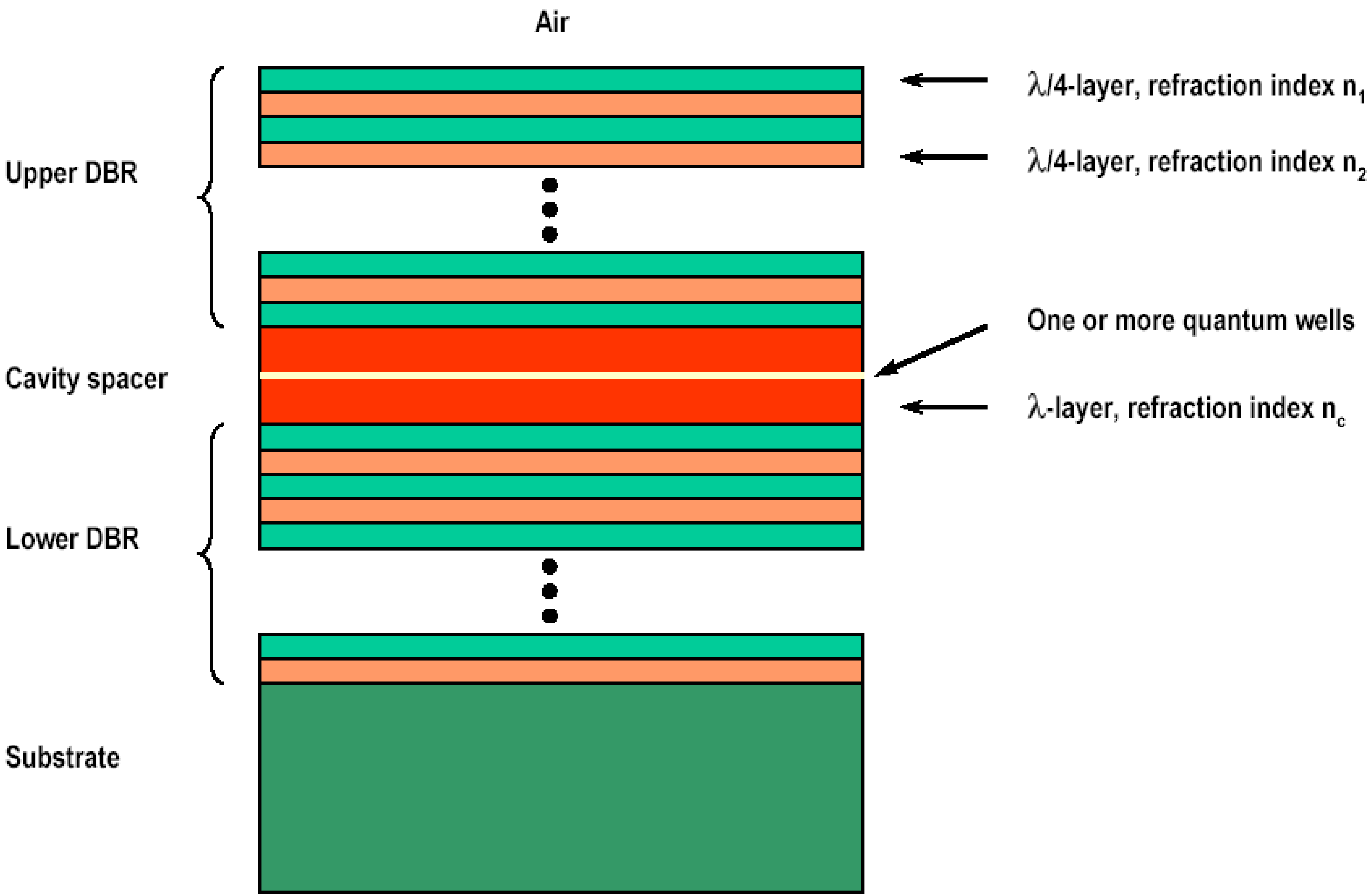
cavity photon:

$$E = \hbar c\sqrt{k_z^2 + k_{\|}^2} = \hbar c\sqrt{(\pi / L)^2 + k_{\|}^2}$$

quantum well exciton:

$$E = E_{gap} - \Delta_{bind} + \frac{h^2 N^2}{2m_r(2L)^2} + \frac{\hbar^2 k_{\|}^2}{2m}$$

Semiconductor microcavity structure



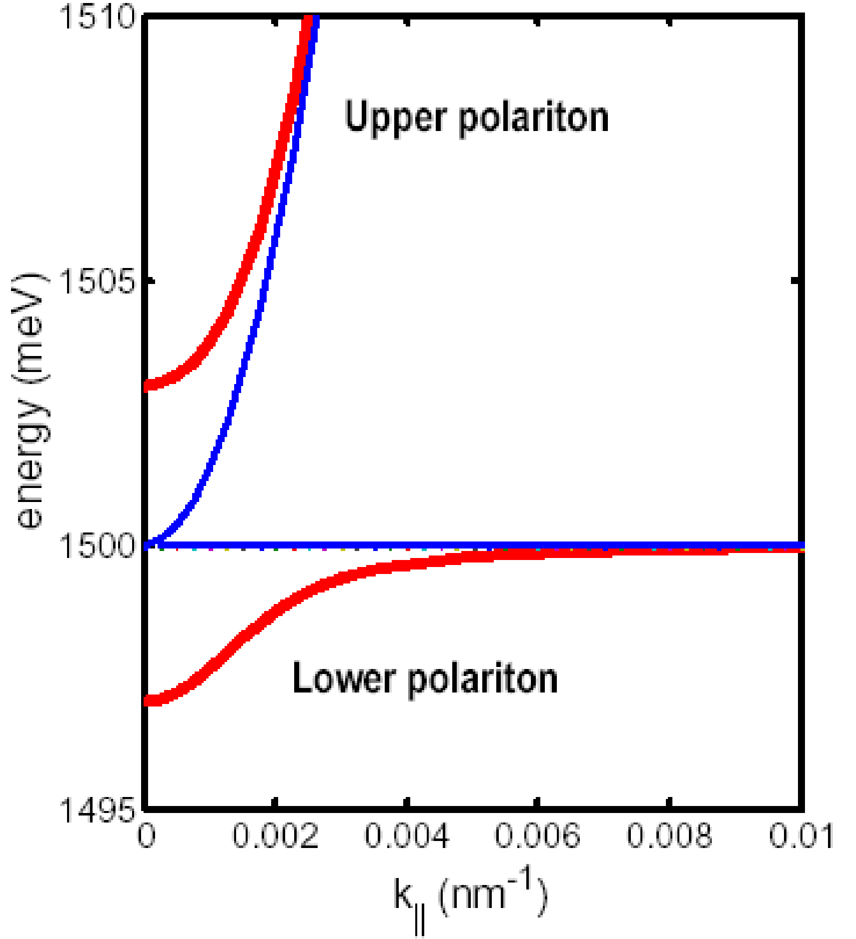
Elementary model of polariton modes

exciton → polariton

photon → polariton

$$H_k = \begin{pmatrix} E_k^{(cav)} & \hbar\Omega_R \\ \hbar\Omega_R^* & E_k^{(exc)} \end{pmatrix}$$

$$E_k^{(\pm)} = \frac{E_k^{(cav)} + E_k^{(exc)}}{2} \pm \frac{1}{2}\sqrt{\left(E_k^{(cav)} - E_k^{(exc)}\right)^2 + 4\left|\hbar\Omega_R\right|^2}$$



Realistic model includes finite polariton lifetime, leaky modes in the mirrors, etc.

P. R. Eastham and P. B. Littlewood, Solid State Communs. 116, 357 (2000).

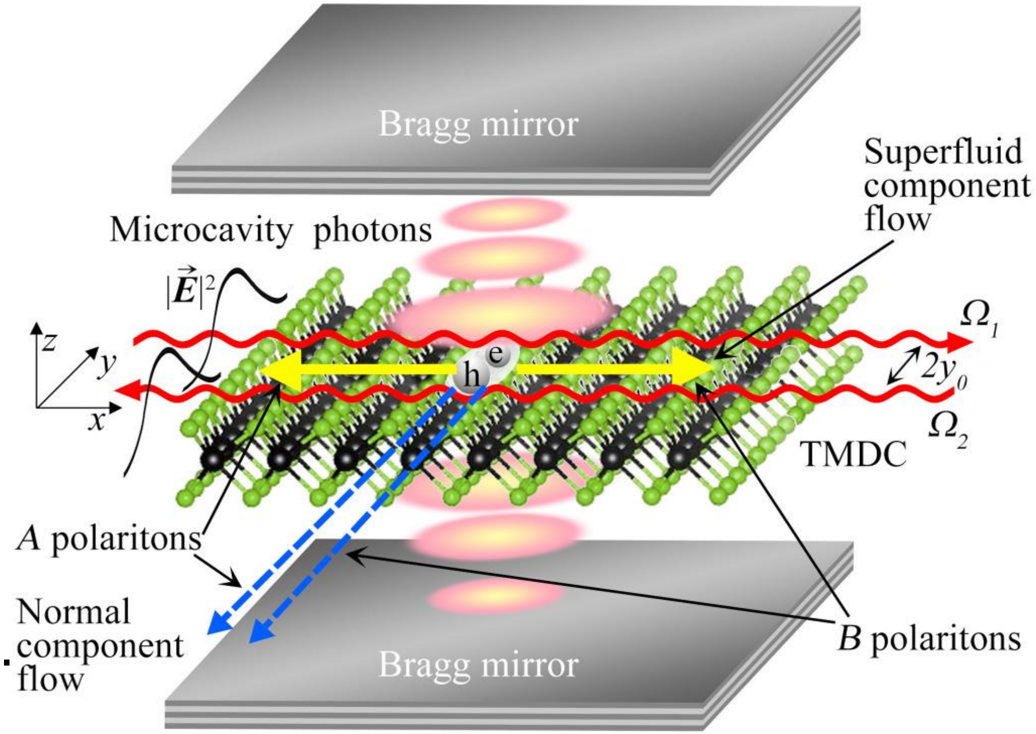
We predict the Spin Hall effect (SHE) for microcavity polaritons in TMDC

1. The polaritons cloud is formed due to the coupling of excitons created in a TMDC layer and microcavity photons.

2. Two coordinate-dependent, counterpropagating and overlapping laser beams in the plane of the TMDC layer interact with a cloud of polaritons.

3. The counterpropagating and overlapping laser beams, characterized by Rabi frequencies Ω_1 and Ω_2 produce the spin-dependent gauge magnetic and electric fields due to strong SOC for electron and holes in TMDC.

Y.-M. Li, et al., Phys. Rev. Lett. 115, 166804 (2015).



4. The gauge magnetic field deflects the exciton component of polaritons consisting from the excitons with different spin states of charge carriers, namely *A* and *B* excitons, towards opposite directions.

5. For the laser pumping frequencies, corresponding to the resonant excitations of one type of excitons (*A* or *B*), the corresponding excitons together with coupled to them photons form polaritons, which deflect to opposite transverse directions.

Using circular polarized pumping, one can excite both *A* and *B* excitons in one valley simultaneously.

G. Wang, et al., Rev. Mod. Phys. 90, 021001 (2018).

6. The normal components of the *A* and *B* polariton flows slightly deflect in opposite directions and propagate almost perpendicularly to the counterpropagating beams.

7. In contrast, the superfluid components of the *A* and *B* polariton flows propagate in opposite directions along the counterpropagating beams.

Hamiltonian of microcavity polaritons in gauge fields

O. L. Berman, R. Ya. Kezerashvili, and Yu. E. Lozovik, Physical Review B 99, 085438 (2019).

The Hamiltonian of TMDC polaritons in the presence of counterpropagating and overlapping laser beams:

$$\hat{\mathcal{H}} = \hat{H}_{\text{exc}} + \hat{H}_{\text{ph}} + \hat{H}_{\text{exc-ph}} + \hat{H}_{\text{exc-exc}}$$

The Hamiltonian of TMDC excitons: $\hat{H}_{\text{exc}} = \sum_{\mathbf{P}} \varepsilon_{\text{ex}}(\mathbf{P}) \hat{b}^{\dagger}_{\mathbf{P}} \hat{b}_{\mathbf{P}}$

$$\varepsilon_{\text{ex}}(\mathbf{P}) = E_{\text{bg}} - E_b + \varepsilon_0(\mathbf{P})$$

E_{bg} is the band gap energy

$$\varepsilon_0(\mathbf{P}) = \frac{(\mathbf{P} - \mathbf{A}_\sigma)^2}{2M}$$

E_b is the binding energy of an exciton

where M is the mass of an exciton and $\mathbf{A}_\sigma$ is the gauge vector potential acting on the exciton component of polaritons, associated with different spin states of the conduction band electron, forming an exciton, $\sigma = \uparrow$ and $\downarrow$.

Hamiltonian of microcavity photons:

$$\hat{H}_{\text{ph}} = \sum_{\mathbf{P}} \varepsilon_{\text{ph}}(P) \hat{a}^{\dagger}_{\mathbf{P}} \hat{a}_{\mathbf{P}}$$

$$\varepsilon_{\text{ph}}(P) = (c/\tilde{n})\sqrt{P^2 + \hbar^2 \pi^2 q^2 L_C^{-2}}$$

L_C is the length of the cavity, $\tilde{n} = \sqrt{\epsilon}$ is index of refraction of the microcavity, q is the longitudinal mode number.

Effective mass of microcavity photons:

$$m_{\text{ph}} = \hbar \pi q / ((c/\tilde{n}) L_C)$$

Hamiltonian of exciton-photon coupling:

$$\hat{H}_{\text{exc-ph}} = \hbar \Omega_R \sum_{\mathbf{P}} \hat{a}^{\dagger}_{\mathbf{P}} \hat{b}_{\mathbf{P}} + \text{H.c.}$$

Ω_R is the Rabi splitting constant

After applying the unitary transformations of the Hamiltonian at $H_{exc\text{-}exc} = 0$ →

The Hamiltonian of lower polaritons:

$$\hat{\mathcal{H}}_0 = \sum_{\mathbf{P}} \varepsilon_{LP}(\mathbf{P}) \hat{p}^{\dagger}_{\mathbf{P}} \hat{p}_{\mathbf{P}}$$

$$\varepsilon_{LP}(\mathbf{P}) = \hbar \pi q L_C^{-1} - |\hbar \Omega_R| + \varepsilon(\mathbf{P})$$

$$\alpha \equiv 1/2(M^{-1} + (c/\tilde{n}) L_C / \hbar \pi q) P^2 / |\hbar \Omega_R| \ll 1$$

$$\varepsilon(\mathbf{P}) = \frac{(\mathbf{P} - \mathbf{A}_\sigma^{(\text{eff})})^2}{2M_p} + V^{(\text{eff})}$$

$\mathbf{A}_\sigma^{(eff)}$ and $V^{(eff)}$ effective vector and scalar potentials, respectively, acting on polaritons,

$$\mathbf{A}_\sigma^{(\text{eff})} = \frac{m_{\text{ph}} \mathbf{A}_\sigma}{M + m_{\text{ph}}}, \quad V^{(\text{eff})} = \frac{A_\sigma^2}{4(M + m_{\text{ph}})}$$

$$M_p = 2\mu, \ \mu = M m_{\text{ph}} / (M + m_{\text{ph}})$$

The dependence of the effective gauge magnetic $B^{(eff)}$ and electric $E^{(eff)}$ fields on the parameter l.

$$\mathbf{B}_{\sigma}^{(eff)} = \nabla_{\mathbf{R}} \times \mathbf{A}_{\sigma}^{(eff)} = \frac{-\eta_{\sigma}\hbar m_{ph}\left(|k_1|+|k_2|\right)}{4l\left(M+m_{ph}\right)}\mathbf{e}_z, \quad \mathbf{E}^{(eff)} = -\nabla_{\mathbf{R}}V^{(eff)} = -\frac{\hbar^2\left(|k_1|+|k_2|\right)^2}{16l\left(M+m_{ph}\right)}\mathbf{e}_y.$$

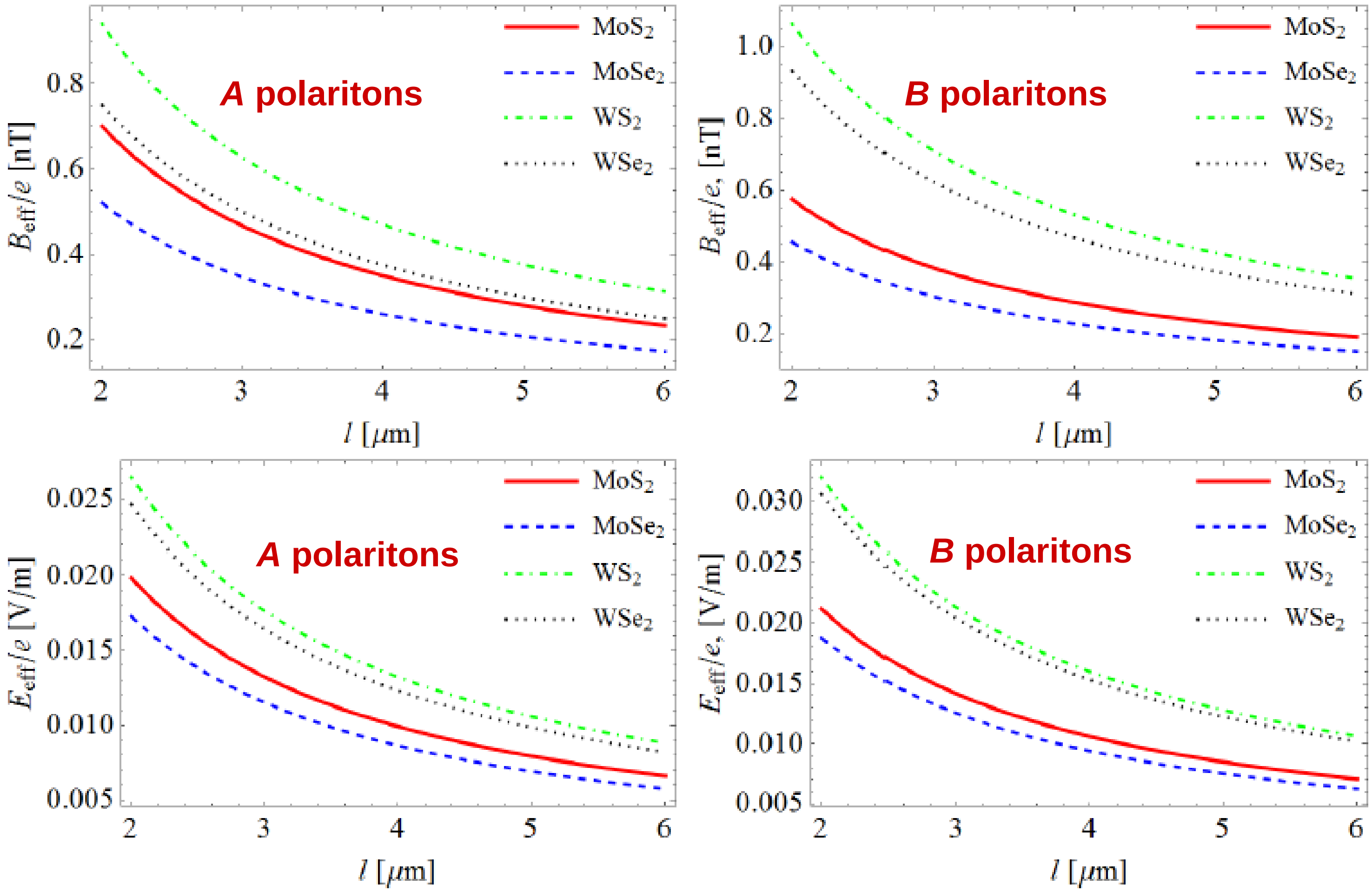
$\eta_{\uparrow} = 1$ for an A exciton and $\eta_{\downarrow} = -1$ for a B exciton.

Calculations performed for $|k_1|+|k_2| = 3\ \mu\mathrm{m}^{-1}$

$l = a^2/8y_0$, $a = 10\ \mu$m is the beam width.

***M* is an exciton mass; m_{ph} is a microcavity photon mass**

y_0 is the spatial shift of two laser beams



Conductivity tensor for non-interacting polaritons

Drude model: $$\frac{d\mathbf{P}}{dt} = \mathbf{E}^{(\text{eff})} + \mathbf{v} \times \mathbf{B}_\sigma^{(\text{eff})} - \frac{\mathbf{P}}{\tau}$$

For a steady state, setting $d\mathbf{P}/dt = 0$,

$$\mathbf{P} = M_p \mathbf{v}$$

$\mathbf{v}$ is the velocity and τ is a scattering time of microcavity polaritons.

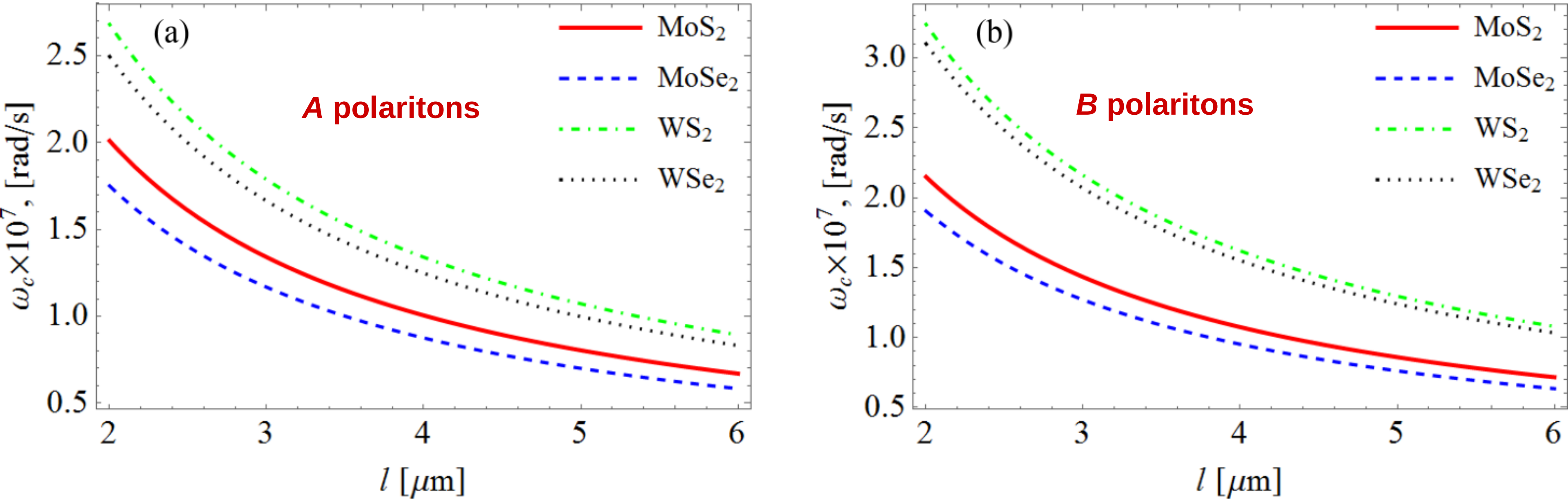
linear polariton flow density is defined as $\mathbf{j} = n\mathbf{v}$ $$\mathbf{E}^{(\text{eff})} = \frac{M_p}{n\tau}\mathbf{j} - \frac{\mathbf{j} \times \mathbf{B}_\sigma^{(\text{eff})}}{n}$$

2×2 resistivity matrix ϱ_σ can be defined as $\mathbf{E}^{(\text{eff})} = \varrho_\sigma \mathbf{j}$,

The conductivity tensor $\tilde{\sigma}_\sigma$ is defined as the inverse matrix to the resistivity matrix ϱ_σ.

$$\sigma_{\sigma xx} = \sigma_{\sigma yy} = \frac{\sigma_0}{1 + \omega_c^2 \tau^2}, \quad \sigma_{\sigma xy} = -\sigma_{\sigma yx} = -\frac{\eta_\sigma \sigma_0 \omega_c \tau}{1 + \omega_c^2 \tau^2},$$

$\sigma_0 = \tau n / M_p$ and $\omega_c = B^{(eff)}/M_p$ is the cyclotron frequency.



$l = a^2/8y_0$, $a = 10\ \mu$m is the beam width. Calculations performed for $|k_1| + |k_2| = 3\ \mu\text{m}^{-1}$

Conductivity tensor for superfluid polaritons

Weakly interacting Bose gas of polaritons in dilute regime below T_c is superfluid

n_n is the concentration of the normal component

n_s is the concentration of the superfluid component

n is the total concentration

$$n_s(T) = n - n_n(T)$$

$$n_n(T) = \frac{3\zeta(3)}{2\pi\hbar^2}\frac{k_B^3 T^3}{c_S^4 M_p}$$

The polaritons in the superfluid component do not collide. $\tau \rightarrow +\infty$.

Drude model: $$M_p\frac{dv_x}{dt} = -\eta_\sigma B^{(\mathrm{eff})} v_y, \quad M_p\frac{dv_y}{dt} = E^{(\mathrm{eff})} + \eta_\sigma B^{(\mathrm{eff})} v_x$$

steady state, which corresponds to $dv_x/dt = dv_y/dt = 0$ $\quad v_y = 0$

the linear superfluid polariton flow density $\mathbf{j}^{(s)} = n_s \mathbf{v}$. $\quad v_x = -E^{(eff)}/\eta_\sigma B^{(eff)}$

conductivity tensor $\tilde{\sigma}_\sigma^{(s)}(T)$ for the superfluid

$$\sigma_{\sigma xx}^{(s)} = \sigma_{\sigma yy}^{(s)} = 0, \quad \sigma_{\sigma xy}^{(s)}(T) = -\sigma_{\sigma yx}^{(s)}(T) = -\frac{n_s(T)}{\eta_\sigma B^{(\mathrm{eff})}}$$

For the conductivity tensor $\tilde{\sigma}_\sigma^{(n)}(T)$ for the normal component

$$\sigma_0(T) = \tau n_n(T)/M_p$$

The sound spectrum of collective excitations: $\epsilon(P) = c_s P$

with the sound velocity c_s

O. L. Berman, R. Ya. Kezerashvili, and Yu. E. Lozovik, Phys. Rev. B, 99, 085438 (2019).

Linear polariton flow density

1. Our calculations show that the contribution to the Hall linear polariton flow density from the superfluid component essentially exceeds the one from the normal component.
2. The contribution to the Hall linear polariton flow density from the superfluid component does not depend on the distance l between the counterpropagating laser beams.
3. The contribution to the linear polariton flow density from the superfluid component in the direction of the effective gauge electric field (y direction) is zero.

At the temperature $T = 300$ K for A polaritons we have obtained the total Hall linear polariton flow density in the presence of superfluidity $j_x^{(tot)} = 8.51887 \times 10^{13}$ nm^{-1}s^{-1} for MoS_2; $j_x^{(tot)} = 9.54342 \times 10^{13}$ nm^{-1}s^{-1} for $MoSe_2$, $j_x^{(tot)} = 9.76334 \times 10^{13}$ nm^{-1}s^{-1} for WS_2, $j_x^{(tot)} = 1.1415 \times 10^{14}$ nm^{-1}s^{-1} for WSe_2.

Proposed experiment

O. L. Berman, R. Ya. Kezerashvili, and Yu. E. Lozovik, Physical Review B 99, 085438 (2019).

1. The proposed experiment for observation of the spin Hall effect for microcavity polaritons is related to the measurement of the angular distribution of the photons escaping the optical microcavity.
2. In the absence of the effective gauge magnetic and electric fields, the angular distribution of the photons escaping the microcavity is central-symmetric with respect to the perpendicular to the Bragg mirrors.
3. We obtain the average tangent of the angles α of deflection for the polariton flow in the (x, y) plane of the microcavity:

$$\overline{\tan\ \alpha} = \left|\frac{j_x}{j_y}\right| = \left|\frac{\sigma_{\sigma xy}}{\sigma_{\sigma yy}}\right|$$

Without superfluidity:

$$\overline{\tan\ \alpha} = \omega_c \tau$$

Proposed experiment for observation of the SHE for microcavity polaritons in the presence of superfluidity

For the normal component:

$$\overline{\tan\ \alpha^{(n)}} = \left|\frac{j_x^{(n)}}{j_y^{(n)}}\right| = \left|\frac{\sigma_{\sigma xy}^{(n)}}{\sigma_{\sigma yy}^{(n)}}\right| = \omega_c \tau$$

For the superfluid component:

$$\overline{\tan\ \alpha^{(s)}} = \left|\frac{j_x^{(s)}}{j_y^{(s)}}\right| = \left|\frac{\sigma_{\sigma xy}^{(s)}}{\sigma_{\sigma yy}^{(s)}}\right| \rightarrow +\infty$$

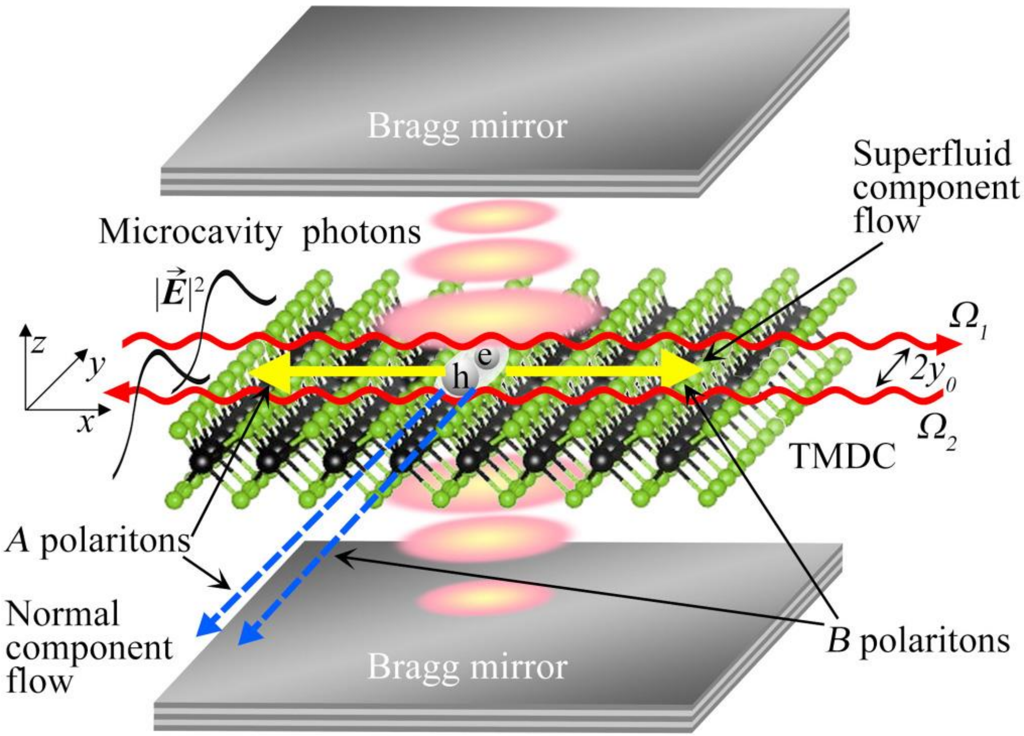
$$\sigma_{\sigma yy}^{(s)} = 0$$

$$\overline{\alpha^{(s)}} = 90^0$$

Calculations performed for $|k_1| + |k_2| = 3\ \mu\text{m}^{-1}$

$l = a^2/8y_0$, $a = 10\ \mu$m is the beam width.

$\overline{\tan\ \alpha^{(n)}} \approx 10^{-5}$, and $\overline{\alpha^{(n)}} \approx 10^{-3\ 0}$



O. L. Berman, R. Ya. Kezerashvili, and Yu. E. Lozovik, Phys. Rev. B, 99, 085438 (2019).

SHE for microcavity polaritons allows to separate the superfluid component from the normal components of the polariton flow!

Conclusions

O. L. Berman, R. Ya. Kezerashvili, and Yu. E. Lozovik, Physical Review B, 99, 085438 (2019).

- We have predicted the spin Hall effect for microcavity polaritons, formed by excitons in a TMDC and microcavity photons.

- We demonstrated that the polariton flow can be achieved by generation the effective gauge vector and scalar potentials, acting on polaritons.

- We have obtained the components of polariton conductivity tensor for both non-interacting polaritons without BEC and for weakly-interacting Bose gas of polaritons in the presence of BEC and superfluidity.

- We demonstrated that due to the SHE the polariton flows in the same valley are splitting: the normal components of the *A* and *B* polariton flows slightly deflect in opposite directions and propagate almost perpendicularly to the counterpropagating beams, while the superfluid components of the *A* and *B* polariton flows propagate in opposite directions along the counterpropagating beams.

- We predicted the method to separate the superfluid component from the normal component of the polariton flow due to the SHE.

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